3D Thermal – Fluidic Topology Optimization

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Introduction

We introduce a pseudo 3D topology optimization framework for coupled conduction/convection problems consisting of a conductive base-layer and a coupled fluid-thermal design layer. Developed with heat sink designs in mind, the resulting framework can be applied to other fluid-thermal design problems where 2D flow effects dominate.

(Ref 1)

Review of 2D Thermal-Fluidic Topology Optimization

(Ref 2, 3)

Lattice Boltzmann Analysis

The lattice Boltzmann method (LBM) is used for both hydrodynamic and thermal analysis. For the hydrodynamic analysis the standard 2D nine distribution function model (D2Q9) is used while the thermal analysis is based on the reduced five distribution function model (D2Q5) illustrated in the figure.

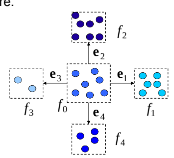
Mass Transport: $f \rightarrow \mathbf{u}, \rho$

$$f_{i,x+\Delta x,t+\Delta t} - f_{i,x,t} = -\frac{\Delta t}{\tau}[f_{i,x,t} - f_{i,x,t}^{eq}] + F$$

Thermal Transport: $g \rightarrow T$

$$g_{i,x+\Delta x,t+\Delta t} - g_{i,x,t} = -\frac{\Delta t}{\tau}[g_{i,x,t} - g_{i,x,t}^{eq}]$$

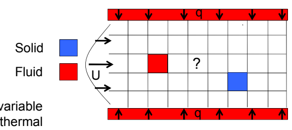
Coupling: $f^{eq}(\mathbf{u}, \rho, T);\, g^{eq}(\mathbf{u}, T)$



Lattice Discretization

Topology Optimization

For topology optimization the analysis problem is augmented through the addition of design variables (**s**).

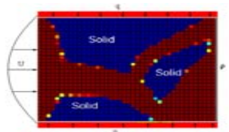


Porosity

Porosity (p) is used as a design variable (p=f(s)) to transition velocity and thermal properties from fluid to solid.

$$u(x,y) = (1 - p(x,y)^{\kappa_v})u(x,y) \quad (1)$$

$$\alpha(x,y) = \alpha_f + (\alpha_s - \alpha_f)p(x,y)^{\kappa_T} \quad (2)$$

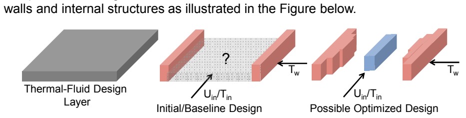


Sensitivity Analysis

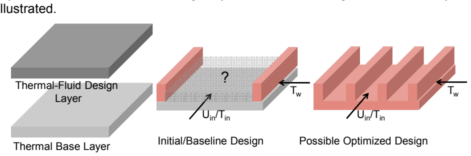
In gradient-based optimization, the design is advanced towards the optimum by finding the derivatives of the design objective with respect to the design variables (s).

Multilayer Thermal-Fluidic Design Optimization

The 2D Thermal-Fluidic Topology Optimization framework introduced by Meyer and Pingen (Refs 2,3) does not permit the creation of heated structures within the computational design domain as there is no thermal connection between the heated walls and internal structures as illustrated in the Figure below.



To overcome this shortcoming and permit the generation of commonly encountered fins as part of the topology optimization process, we propose the use of a dual/multi-layer model that includes a design layer and a conducting thermal base-layer as illustrated.



3D Analysis and Optimization Framework

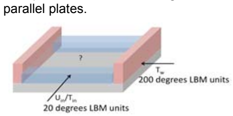
For the 3D approach we use the D3Q7 thermal LBM model. In order to permit/prohibit heat-transfer from the base-layer, the heat transfer across layers must depend on the design variables, leading to the following equations

$$\tilde{g}_5^2(t,x) = g_6^2\left(1 - p(x)^{\kappa_p}\right) + g_5^1\left(p(x)\right)^{\kappa_p}$$
$$\tilde{g}_6^1(t,x) = g_5^1\left(1 - p(x)^{\kappa_p}\right) + g_6^2\left(p(x)\right)^{\kappa_p} \quad (3)$$

Through Eqs (1-3), the design variables have 3 explicit effects on the physical model, and add further complexity to the sensitivity analysis.

Example Problem Definition

The figure below illustrates our first design example. Blue represents prescribed fluid regions to reduce inlet and outlet effects. Red represents prescribed solid material. Flow is driven by a pressure difference between the inlet and outlet. The objective is to maximize the heat transfer from the thermal base-layer. Analytically, an optimal number of straight fins can be determined assuming flow between parallel plates.

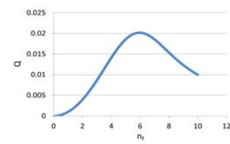


Heat Transfer as a Function of Fins - n

$$Q = \frac{\Delta P W^3 H c_p (T_w - T_i)}{12 \nu L n^2}\left(1 - e^{-\frac{6\alpha L^2 \mu n^4 Nu}{W^4 \Delta P}}\right) \quad (4)$$

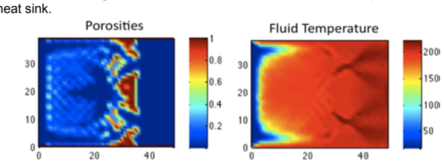
The figure on the right shows that having approximately 6 fins represents the optimal fin/channel configuration for a particular choice of parameters.

Note: fin thickness was neglected for this analysis.



Initial Results: General Topology Optimization

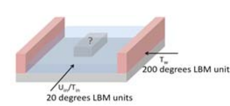
Applying the developed optimization framework to the initial design domain shown above while allowing each point in the domain to vary independently from fluid to solid, leads to a large amount of intermediate porosities as shown below, resulting in a non-intuitive, impractical and non-physical design. Ultimately, the desired outcome is a design with no intermediate porosities in order to produce a realistic heat sink.

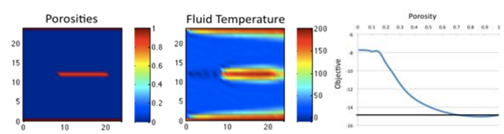


Testing and Improving the Problem Formulation

Single Fin Test

Instead of a design domain with design variables corresponding to each lattice node, we simplified the problem to the creation of one fin in the center of the domain. The results are shown in the figures below.

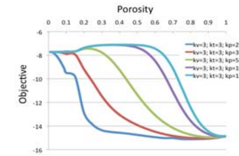




Testing and Improving the Problem Formulation

Scaling Factor Investigation

The porosity vs. heat flux graph must be continuously decreasing. Changes to the scaling factors in Equations (1,2,3) were investigated to change the functional dependency of the hydrodynamic and thermal parameters on the design variables.

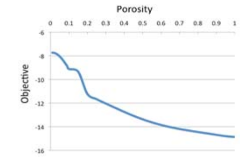


Penalty Formulation

Since combinations of different scaling factors did not produce the desired result, we turned to a penalty formulation to discourage intermediate porosities.

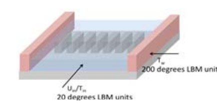
$$F_p = F + C\left(p^f\right)\left(1 - p^f\right) \quad (5)$$

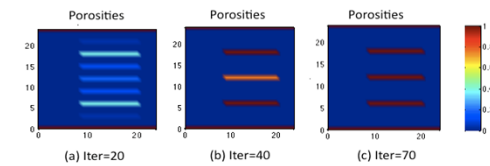
Using κ_v=2, κ_t=4, and κ_p=2, and a penalty formulation with C=2 and f=2, leads to a continuous decrease in the objective as a function of the design variable for a single fin.

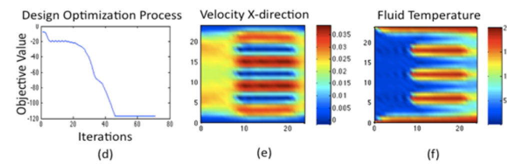


Multi-Fin Optimization Results

As shown previously in Equation 4, there is a maximum amount of fins for design domains. The design domain tested allowed for the creation of seven possible fins but created three fins.







Discussion and Conclusions

We have introduced a lattice Boltzmann method based pseudo 3D fluid-thermal topology optimization approach. Differing from past LBM based topology optimization, it was shown that the non-linear nature and complex interaction of hydrodynamic and thermal effects necessitates the use of a penalty formulation in order to prevent intermediate porosities. The resulting optimization framework was applied to a multi-fin heat sink optimization. Future work will focus on the generalization of the presented work.

References

1. http://www.legitreviews.com/article/1202/2/. Heat sink picture.
2. Pingen, G., and Meyer, D., 2009. "Design optimization for thermal transport." In Proceedings of the ASME 2009 Fluids Engineering Summer Meeting, August 2-5, 2009, Vail, Colorado, FEDSM2009-78408.
3. Meyer, D., 2010. "Lattice boltzmann thermal convective topology optimization." Master's thesis, Department of Mechanical and Aerospace Engineering, University of Colorado at Colorado Springs, October.

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