# Mathematics in the Natural World

By
Jenny Middleton

# OUTLINE

- I. Mathematics and the Beginnings of Civilizations
- II. Ongoing Question of Abstraction
- III. Finality of Abstraction
- IV. NATURE: A Continual Portrait of Mathematics

# I.

# Mathematics and the Beginnings of Civilization

## Babylonians

- Concept of angle
- Crude calculations areas of fields
- Division of fields

## Egyptians

- Herodotus, Egyptian geometry, and flooding of Nile

# II.

# Ongoing Question of Abstraction

# Philosophers

- Pythagorean
  - abstractions vs. physical objects
- Eleatic
  - discrete and continuous
- Sophist
  - understand universe
- Platonist
  - distinction of numbers
  - ideal and material
- Eudoxus
  - proof of shapes

# Greeks

- Desire to understand world
- Math as an investigation of nature
- Reduction of chaos and mystery

# Renaissance

- Math as one remaining body of truth
- Unity of God's view of nature and mathematic's view of nature
- Contribution of concepts

# 17th Century

- Investigation of nature
- Union of mathematics and science

# 18th Century

- Math as means to physical end
- Design of universe

# III.

# Finality of Abstraction

# 19th Century

- Concepts with no direct physical meaning
- Arbitrary concepts not physical yet useful
- Creation of own concept's in mathematics

"Whereas in the first part of the century they accepted the ban on divergent series on the ground that mathematics was restricted by some inner requirement or the dictates of nature to a fixed class of correct concepts, by the end of the century they recognized their freedom to entertain any ideas that seemed to offer any utility."

Morris Kline

# IV.

# NATURE

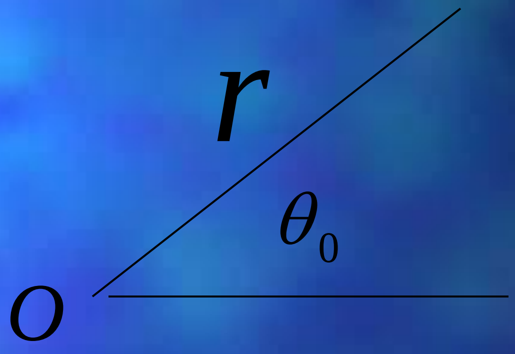
# A Continual Portrait of Mathematics

# NATURE

- **Calculus**
  - polar coordinates
- **Geometry**
  - tiling by regular polygons
- **Abstract Algebra**
  - group theory and symmetry
- **Fractals**
  - self-similarity

# Calculus

- $(r, \theta_0)$




- r = a — circle of radius a centered at 0
- $\theta = \theta_0$ — Line through 0 making an angle with initial ray $\theta_0$

# *Calculus*

- $X = R \cos 0$
- $\frac{X}{R} = \cos 0 = \frac{\text{adjacent}}{\text{hypotenuse}}$

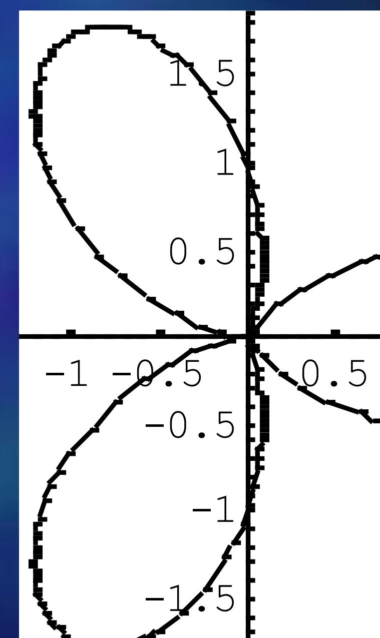
- $Y = R \sin 0$
- $\frac{Y}{R} = \sin 0 = \frac{\text{opposite}}{\text{hypotenuse}}$

# *Calculus*

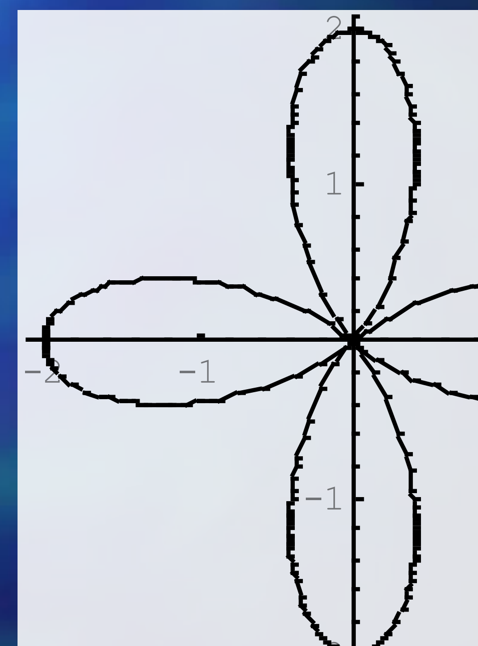
- Mathematica program

- Needs["Graphics`Graphics"]
  PolarPlot[(1 + Cos [5t]),{t, 0, 2Pi}]

# PolarPlot



PolarPlot 1+Cos 4 t t 0



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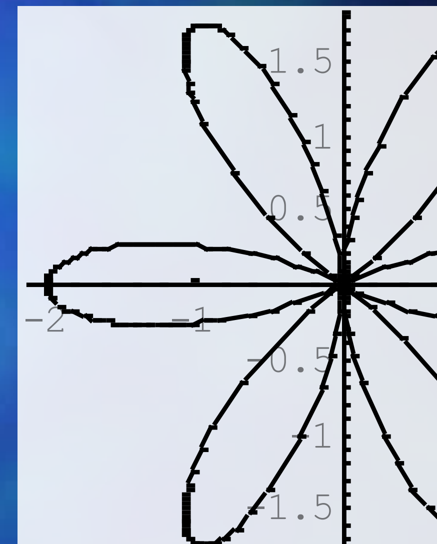
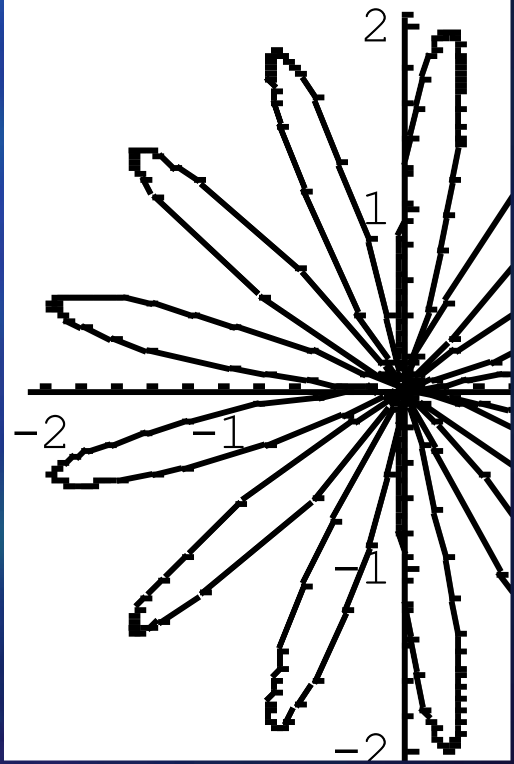
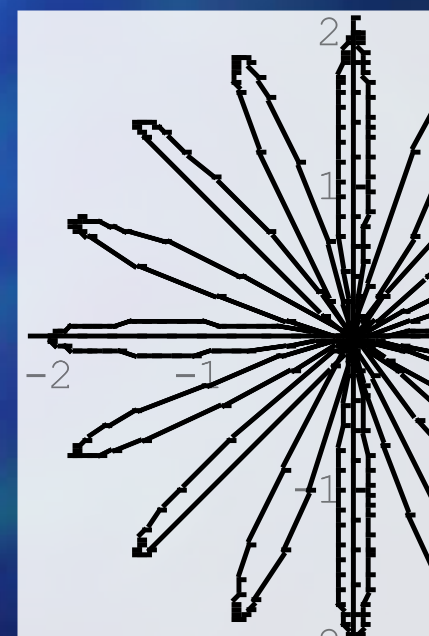


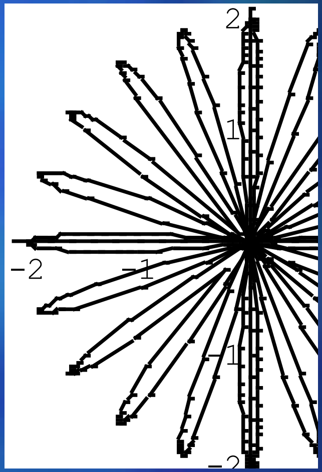

PolarPlot

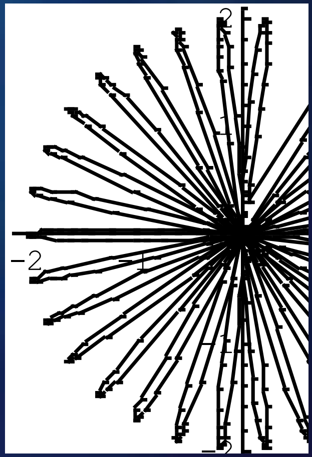


PolarPlot 1+Cos 16t t 0

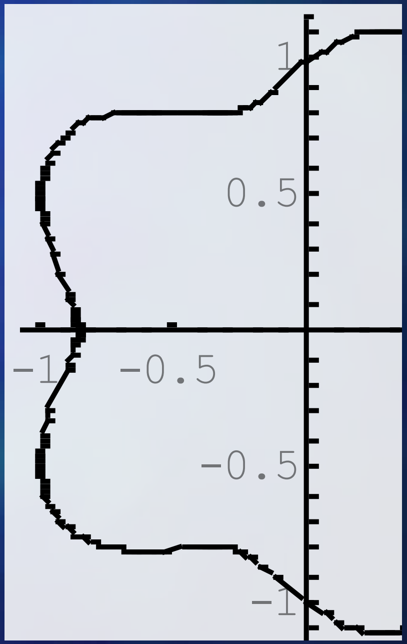


PolarPlot



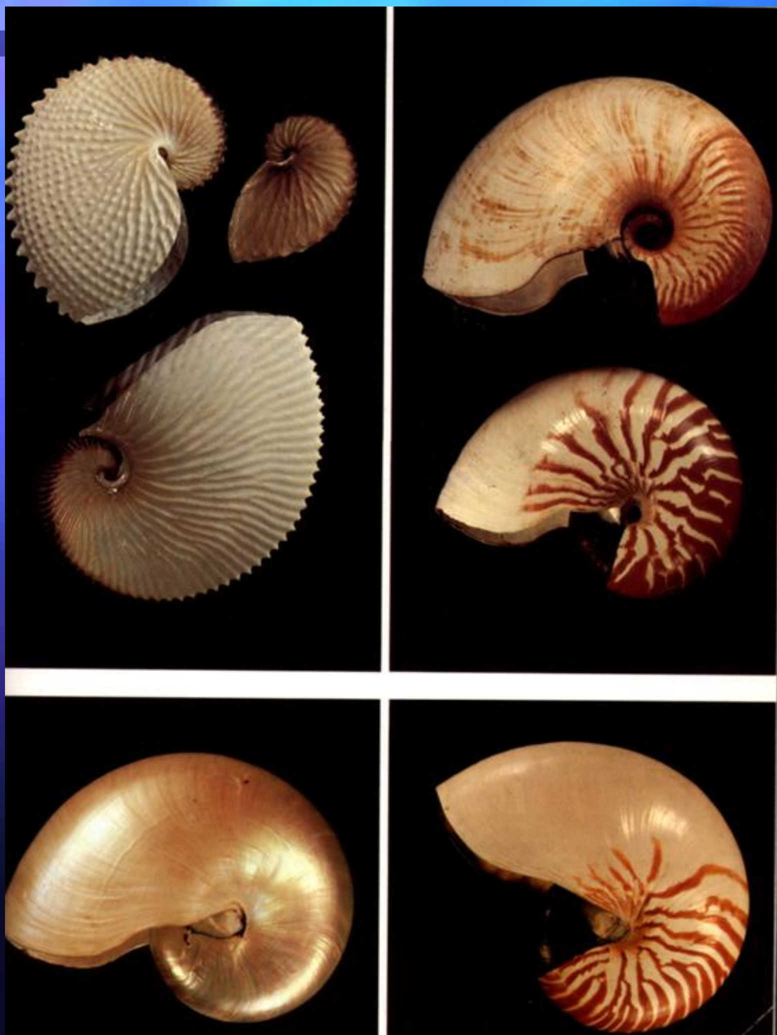



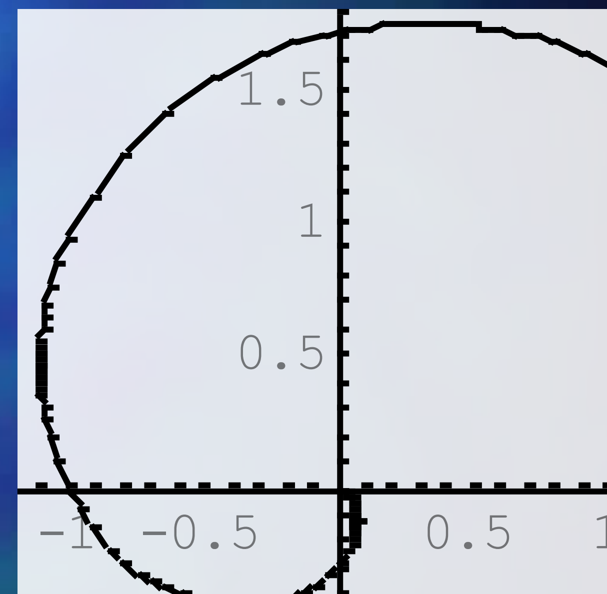
PolarPlot 1+ 15 Cos 5 t t 0



PolarPlot 1+ 5Cos 5 t t 0

PolarPlot





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  ,   , 12 Pi  , Asp ctR  tio -> Automatic,
 PlotPoints -> 80
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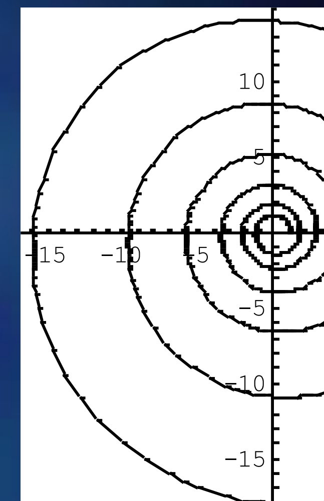

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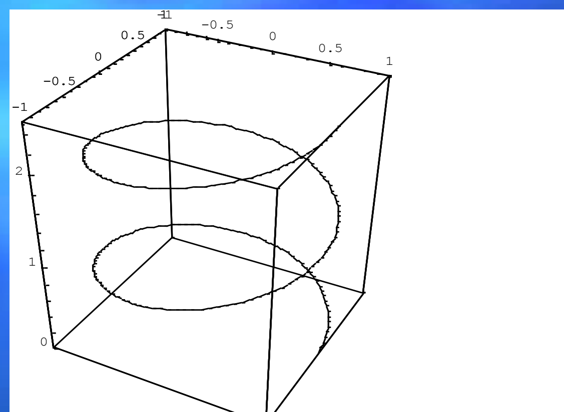
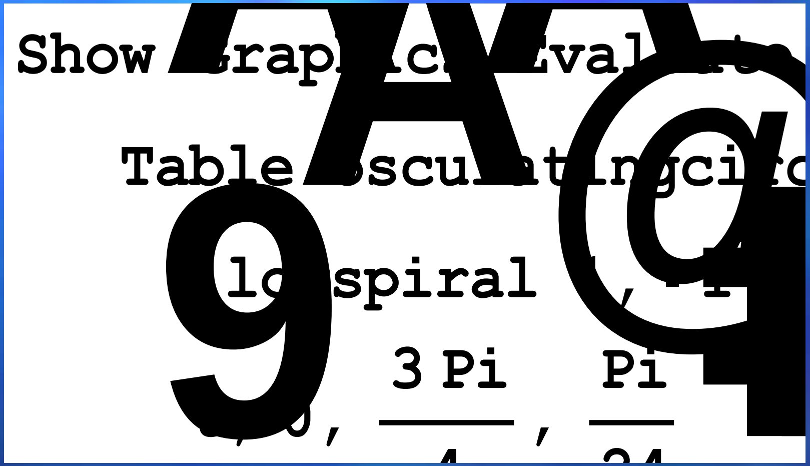
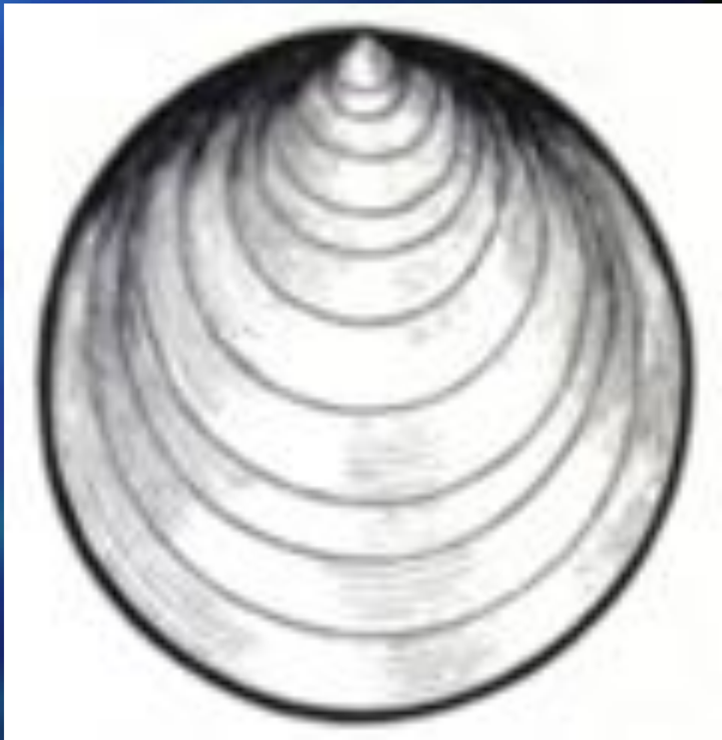
helix ... cos ... sin ...
ParametricPlot3D[Evaluate[helix ... .2
..., 4 Pi , PlotPoints -> 20 ,
PlotRange -> ... 1 ... 1 ... 8
```




Show
Table
spiral
3 Pi
Pi



# Abstract Algebra

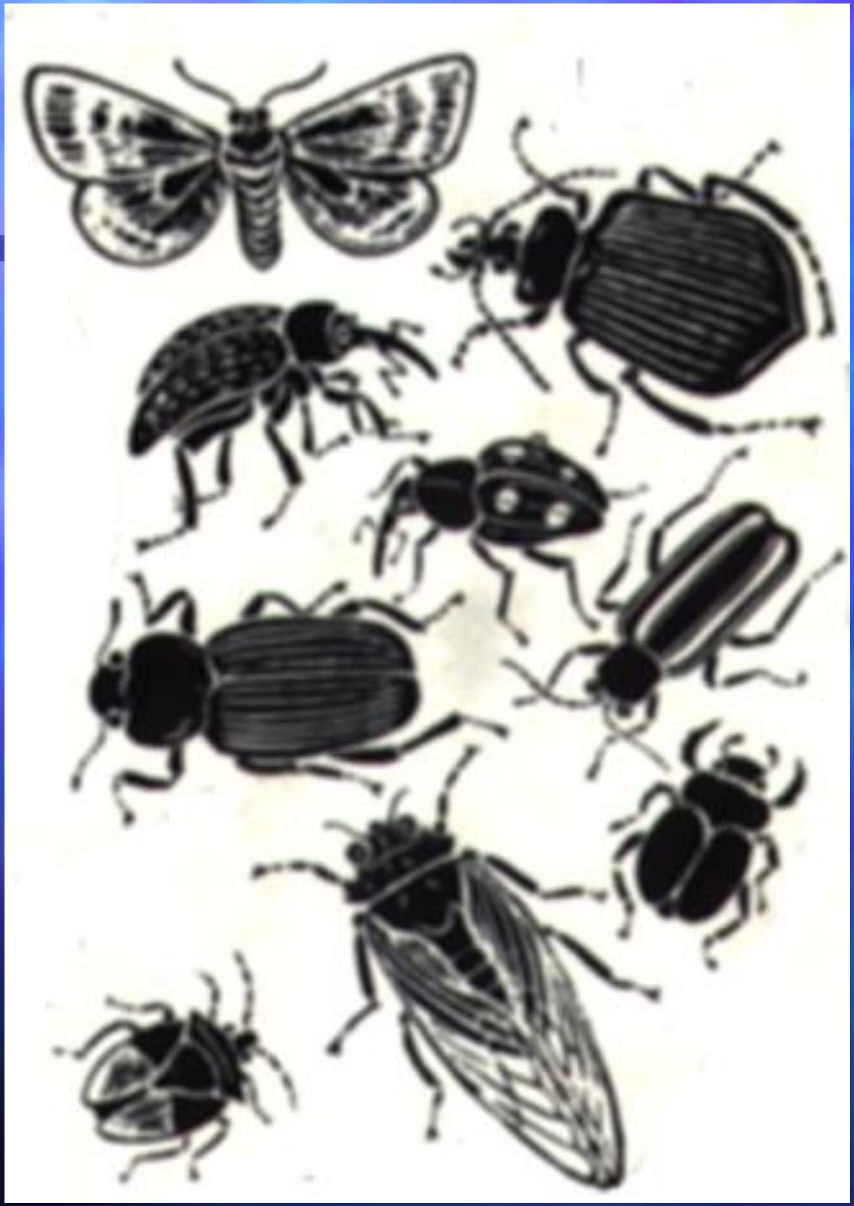
- Dihedral group of order n = $D_n$
- Elements can include
  - flip horizontally
  - flip vertically
  - flip diagonal
  - rotation
- 4 sides = 8 elements
  3 sides = 6 elements
  n sides = 2n elements

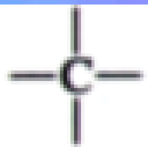
# *Abstract Algebra*

- Because of the similarity of these objects to the group $D_n$ , properties of closure, inverse, indentity, and  associativity hold.
- We may also examine the object with terms such as:
  - subgroups, center of group, centralizer of group, cyclic group, generator, permutations, cosets, isomorphism, Lagrange's theorem, direct products, normal subgroups, homomorphisms, etc.

# Abstract Algebra

- Many objects have rotational symmetry and not reflective symmetry.  If so, they are called cyclic rotation groups of order n.
- Some have only reflective symmetry and no rotational symmetry (except $R_0$ ) and I have chosen to call them reflection group.



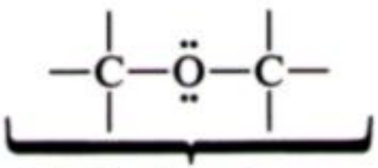
Carbon atoms

Oxygen atoms

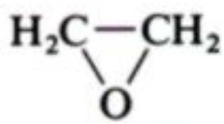
Hydrogen and halogen

$H_2NCH_2CH_2CH_2CH_2NH_2$

Putrescine
(found in decaying meat)

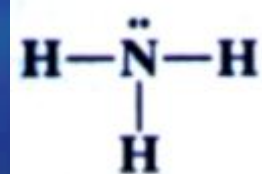


The functional group
of an ether

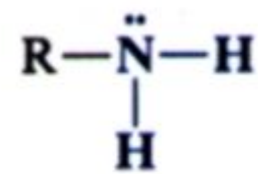


Ethylene
oxide

Tetrahydrofuran
(THF)

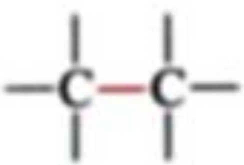


Ammonia



(an amine)

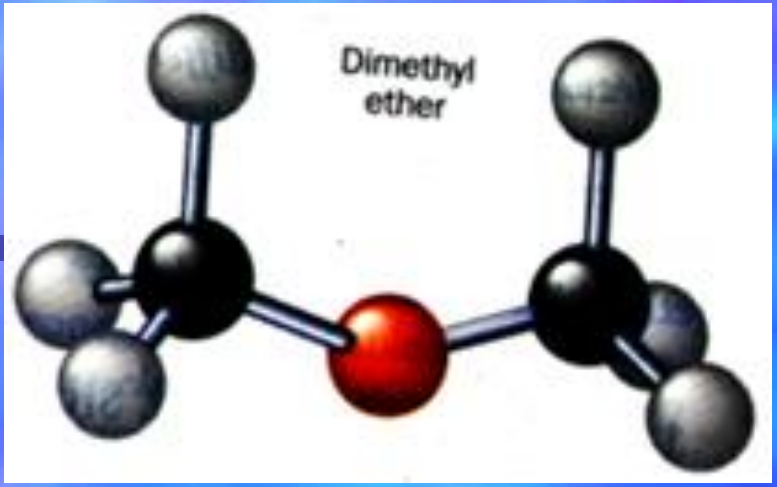
Carbon–carbon bonds

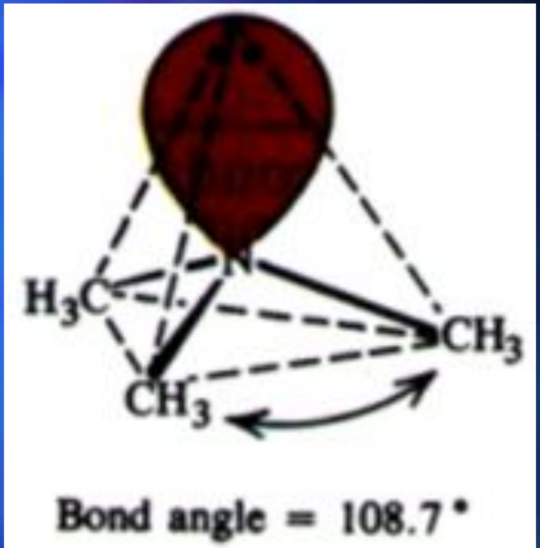


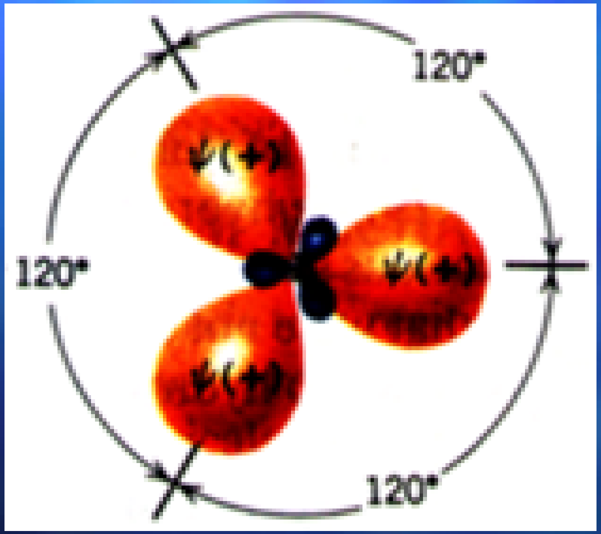
Single bond

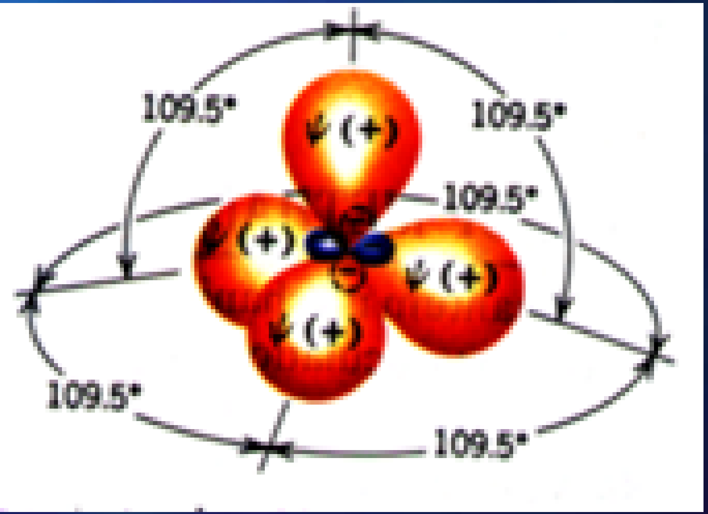
Double bond

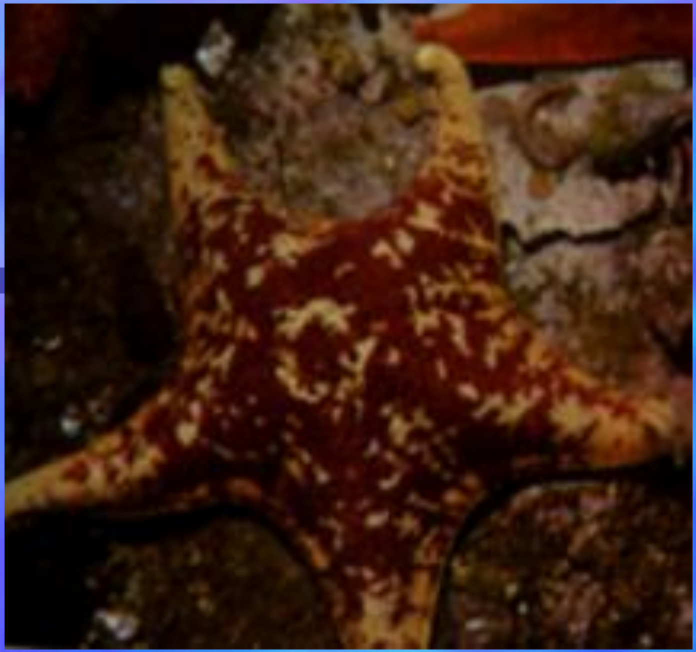
Triple bond

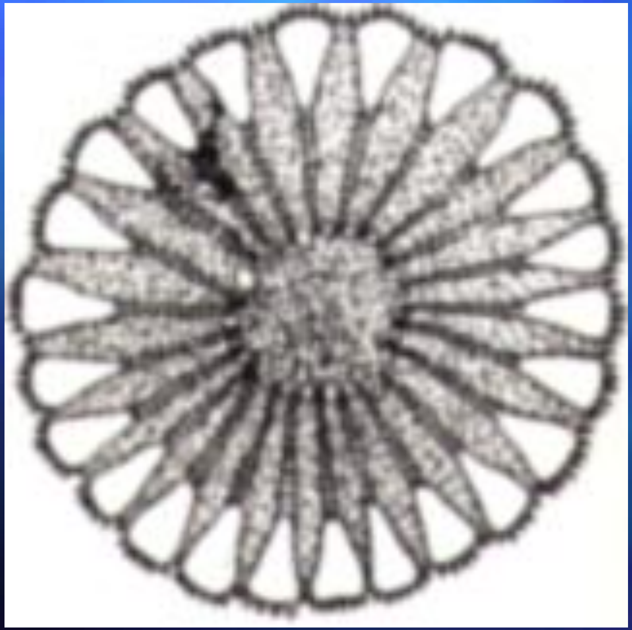

Dimethyl
ether

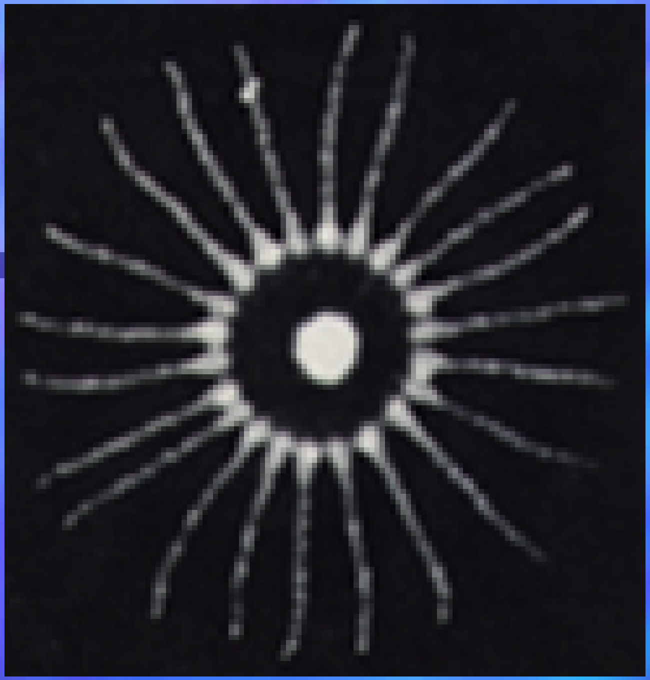

H3C
CH3
CH3
Bond angle = 108.7°

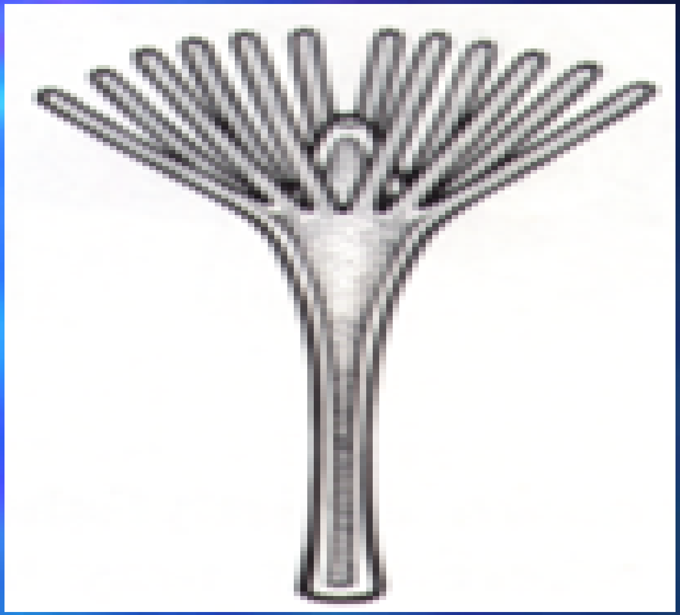

120°
120°
120°

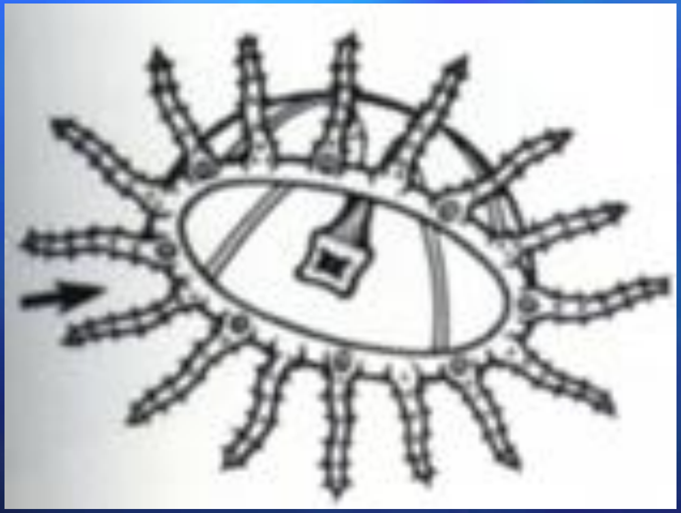

109.5°
109.5°
109.5°
109.5°
109.5°

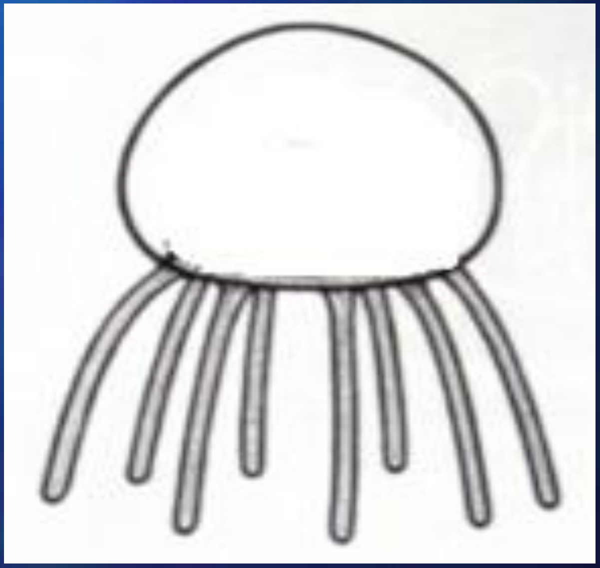


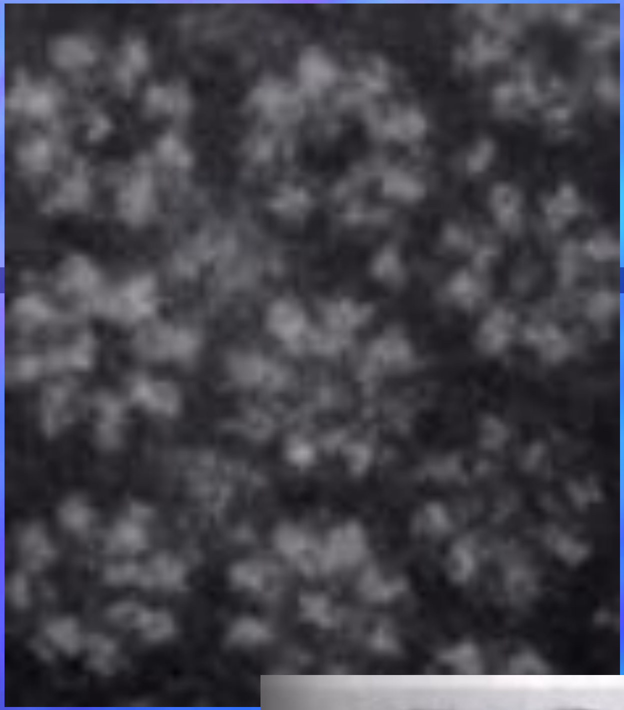



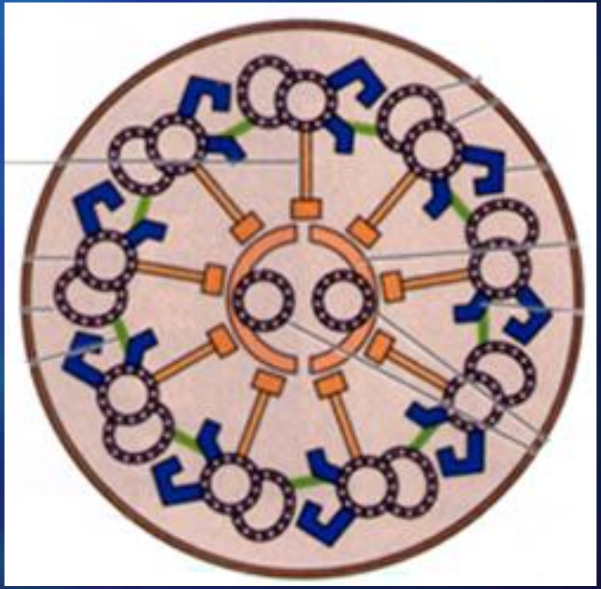











# *Geometry*

- Any polygon can be inscribed in a circle
- Sum of angle of adjoining triangles must be 360
- Sum of interior angles of an n-gon is (n-2)Pi
- Equation for one interior angle is

$$\frac{(n-2)Pi}{n} = \frac{n\ Pi - 2Pi}{n} = Pi - \frac{2Pi}{n}$$

# Geometry

- N=3 Pi/3 x 6 = 2Pi
- N=4 Pi/2 x 4 = 2Pi
- N=5 3Pi/5 x 4 = 12Pi/5 doesn't = 2Pi
- N=6 2Pi/3 x 3 = 2Pi
- N > 6 one angle must be > 2Pi/3 and add up to 2Pi (less than 3 times)
- only 2,1 times left / angle Pi, 2Pi
  BUT not an n-gon

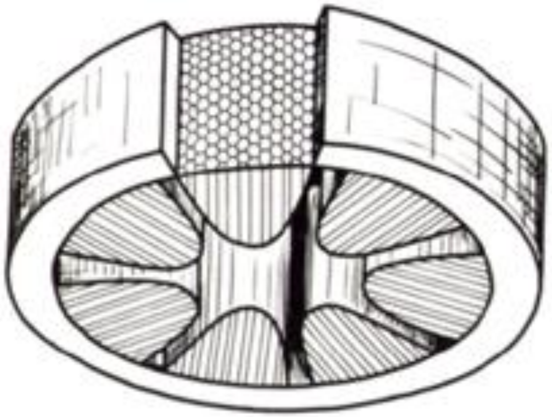
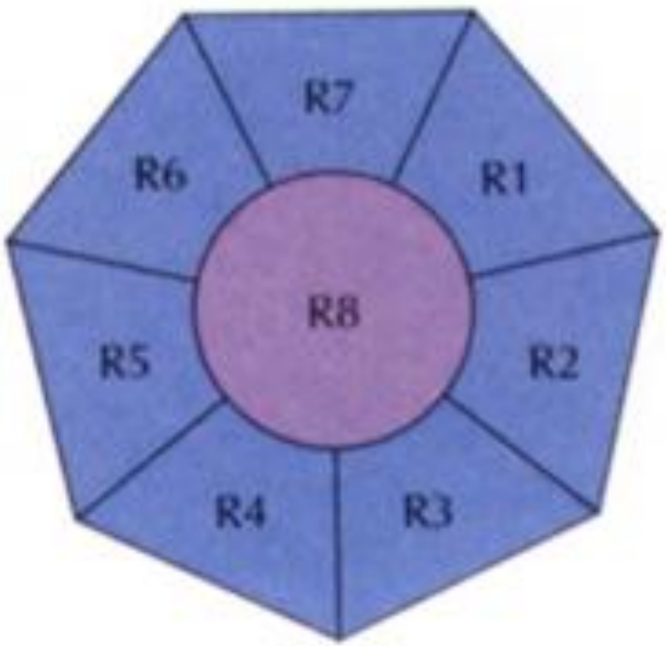
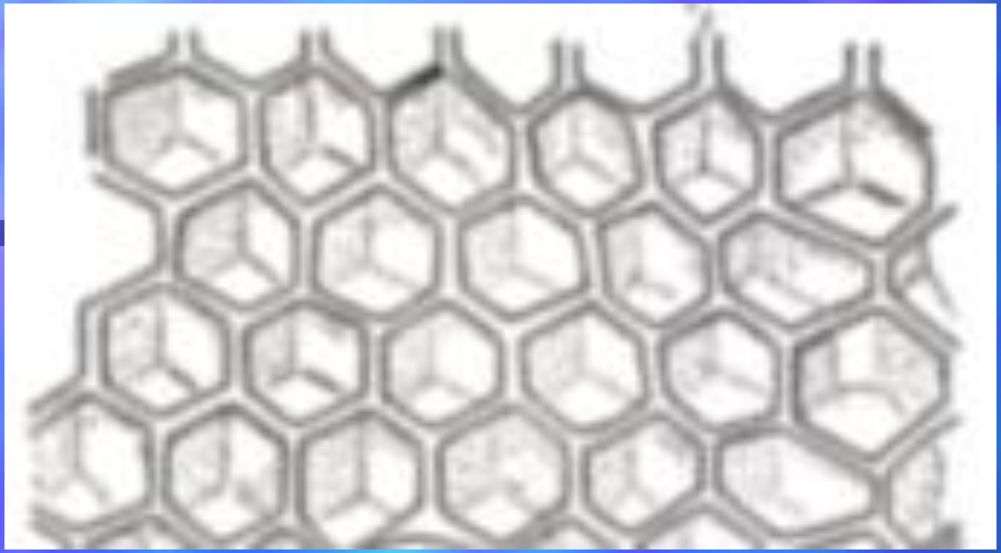
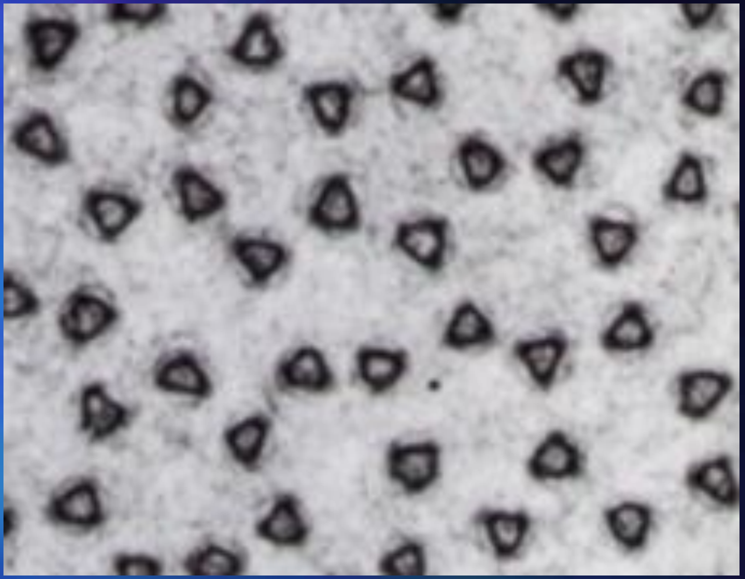
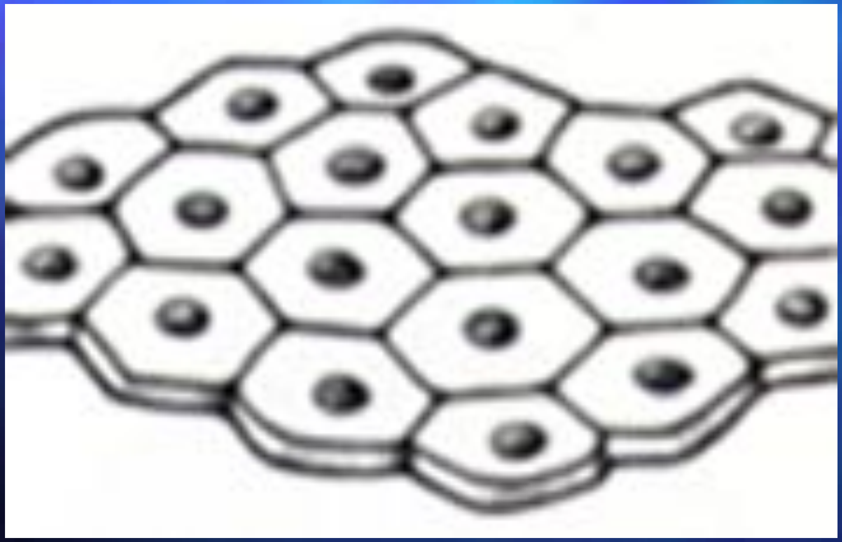
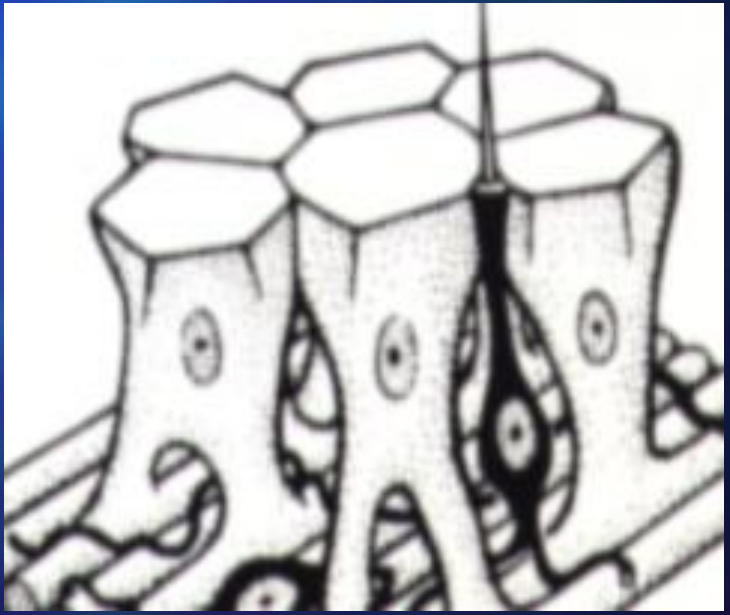


*Figure 70.* Cross-sectional slice cut from an ommatidium of a fly's eye, showing the radial arrangement of the fine structure in the rhabdomeres.



# *Fractals*

- Fractals are objects with fractional dimension and most have self-similarity.
- Self-similarity is when small parts of objects when magnified resemble the entire way.
- The boundaries are of infinite length and are not differentiable anywhere (never smooth enough to have a tangent at a point).

# *Fractals*

- One specific class of fractals is trees.
- Fine-scale structures of the tiniest twig are similar to that of the largest branches.

# Fractals

- Definition 5.1.2
  - If an object can be decomposed into N subobjects, each of which is exactly like the whole thing except that all lengths are divided by s, then the object is exactly self-similar, and the similarity dimension d of the object is defined by $d = \frac{\log N}{\log s}$. ________ ________________

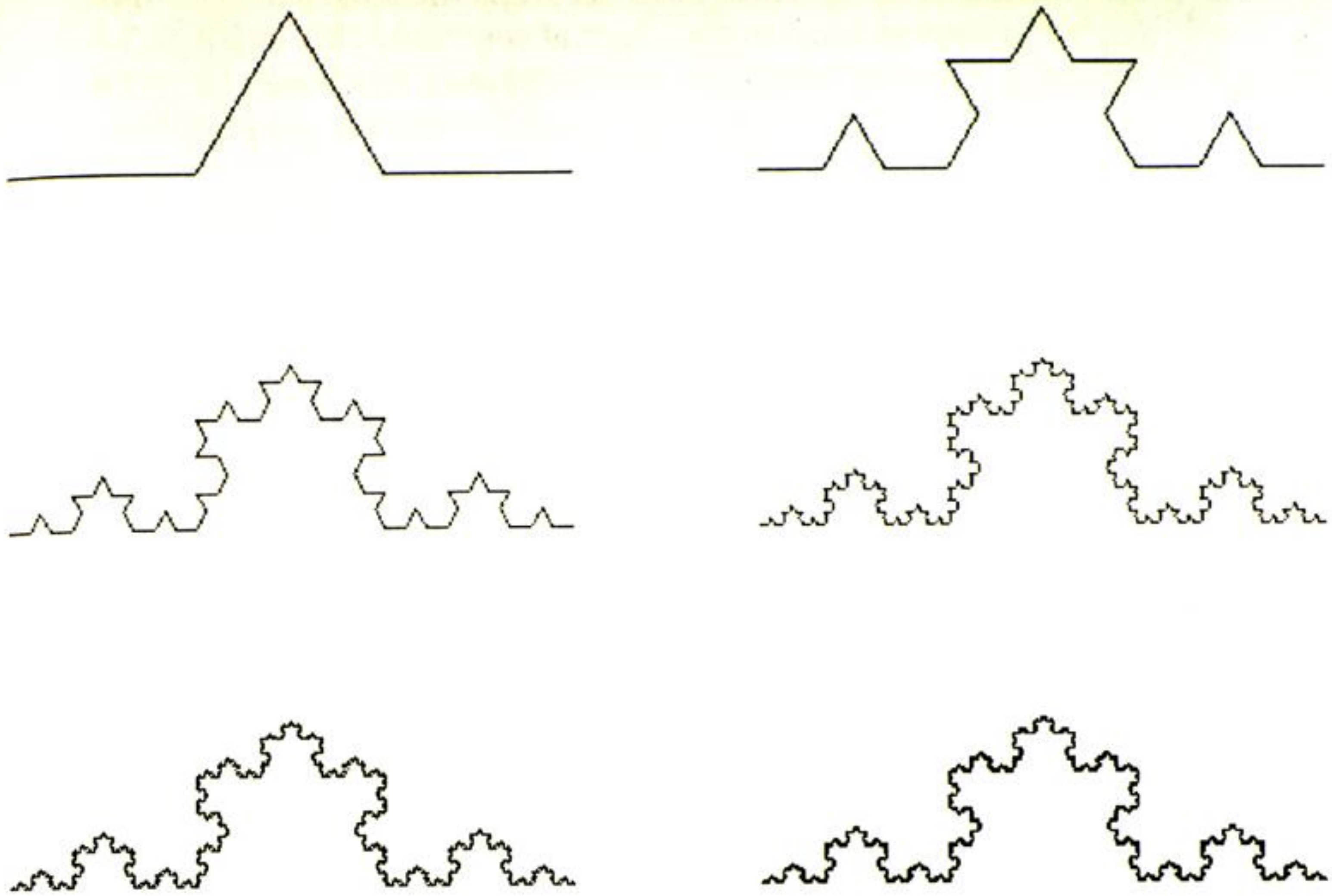


Figure 5.1. First six steps of the Koch curve.



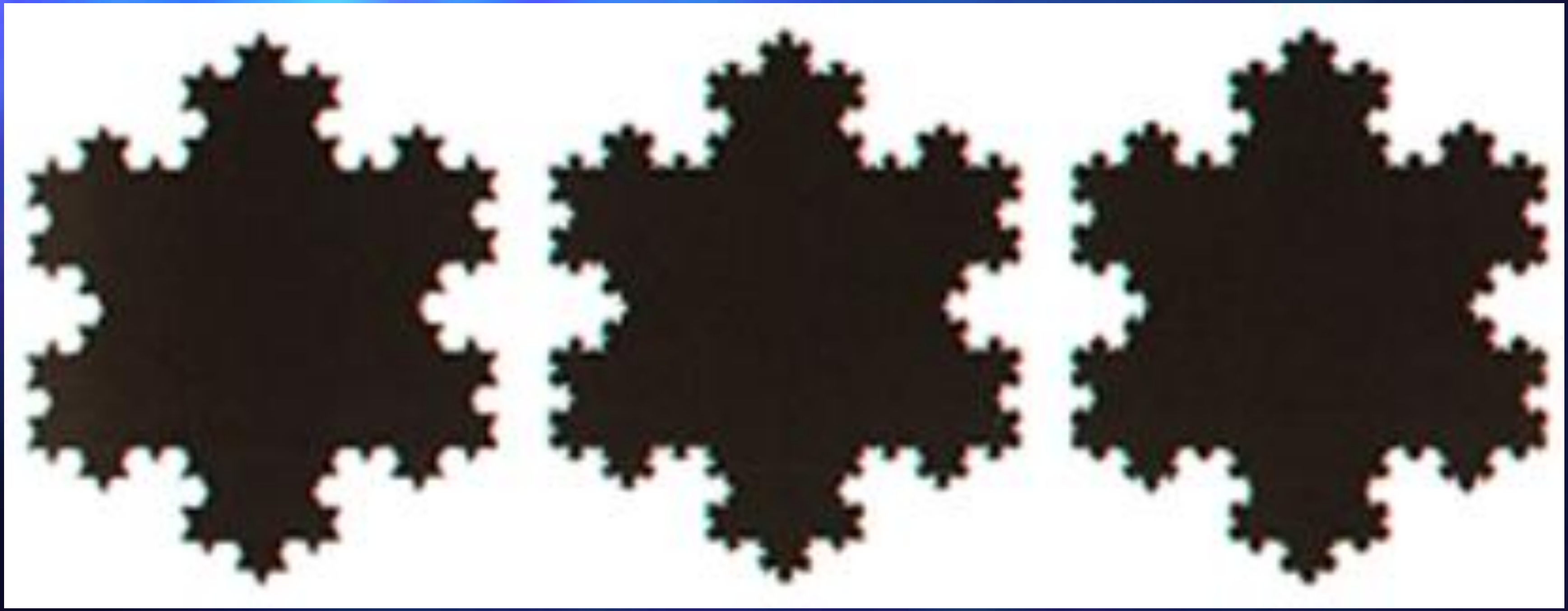
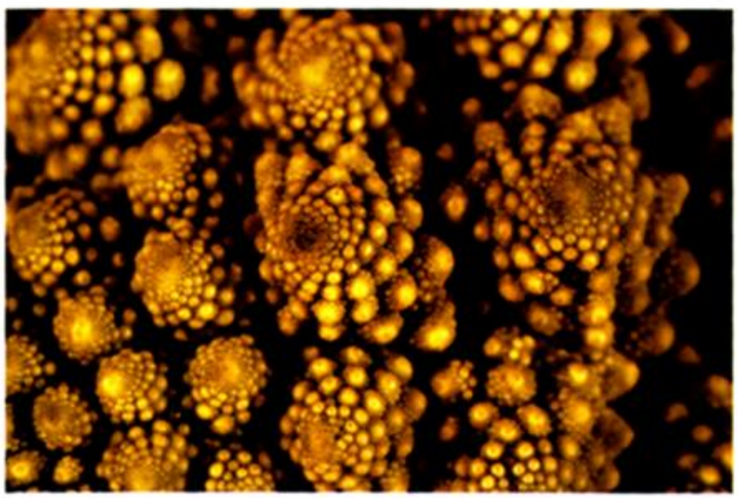
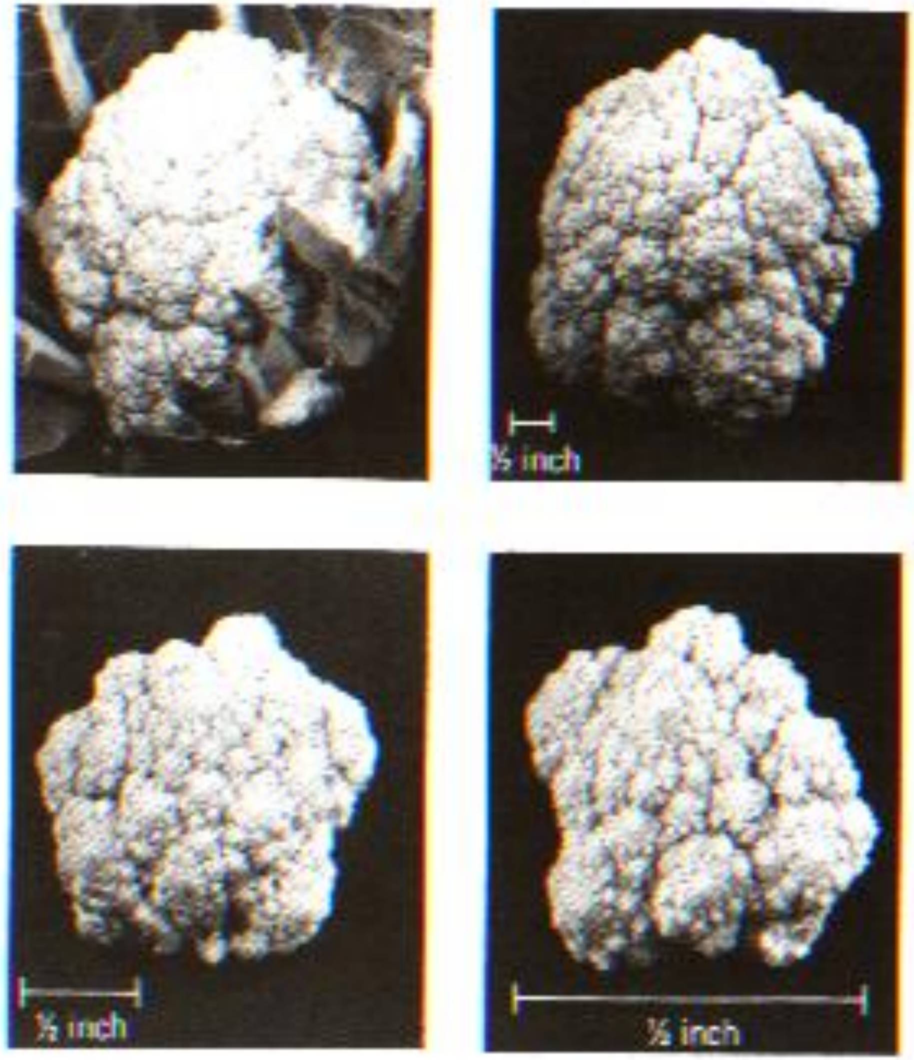
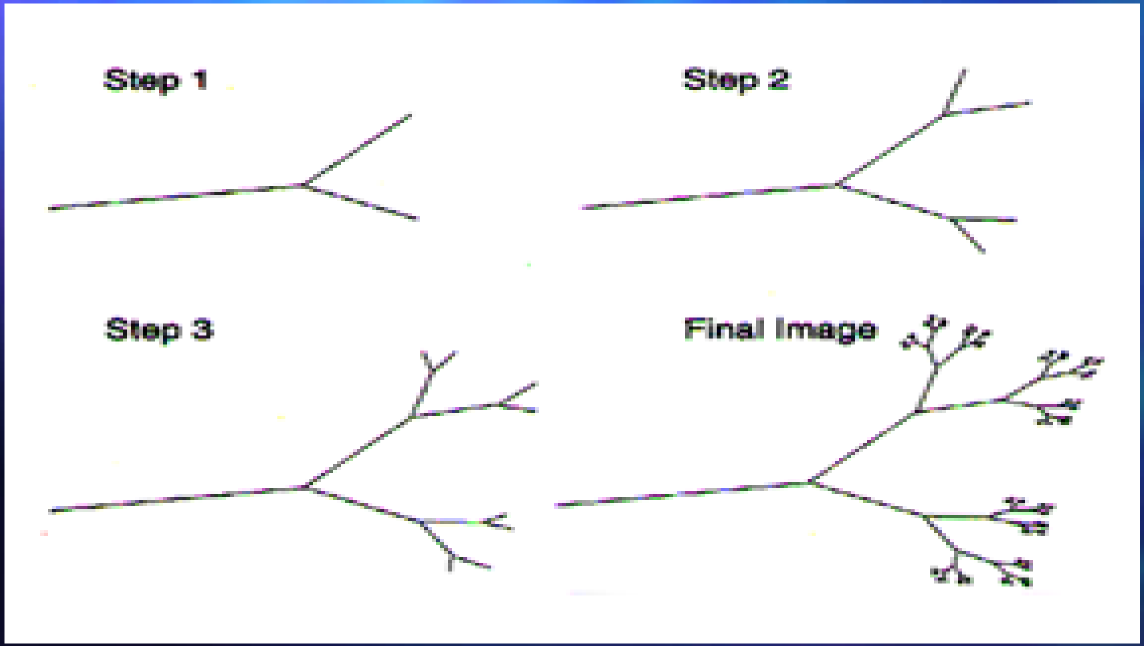


Plate 3: Broccoli Romanesco.






Step 1
Step 2
Step 3
Final Image