Epidemic Models

Beverly Lewis

MODEL

MODEL

Model Cars

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Model Cars

Architectural Models

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Architectural Models

Super Models

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For example:

A model car has a hood, tires, and doors.

A real car has a hood, tires, and doors.

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For example:

A model car has two doors that are between two fenders.

A real car has two doors that are between two fenders.

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Every aspect of the object being modeled in also an aspect of the model.

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For example:

A real car has an engine that runs.

A model car does not have an engine that runs.

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For example:

A real car has leather seats with tan seat belts.

A model car has cloth seats with black seat belts.

Kermack -McKendrick Model

or

SIR Model

Three Classes for the SIR Model

Susceptible

Infected

Removed

The following are the assumptions made by the SIR Model.

The increase in the number of the infective class is “at a rate proportional to the number of infectives and susceptibles.”

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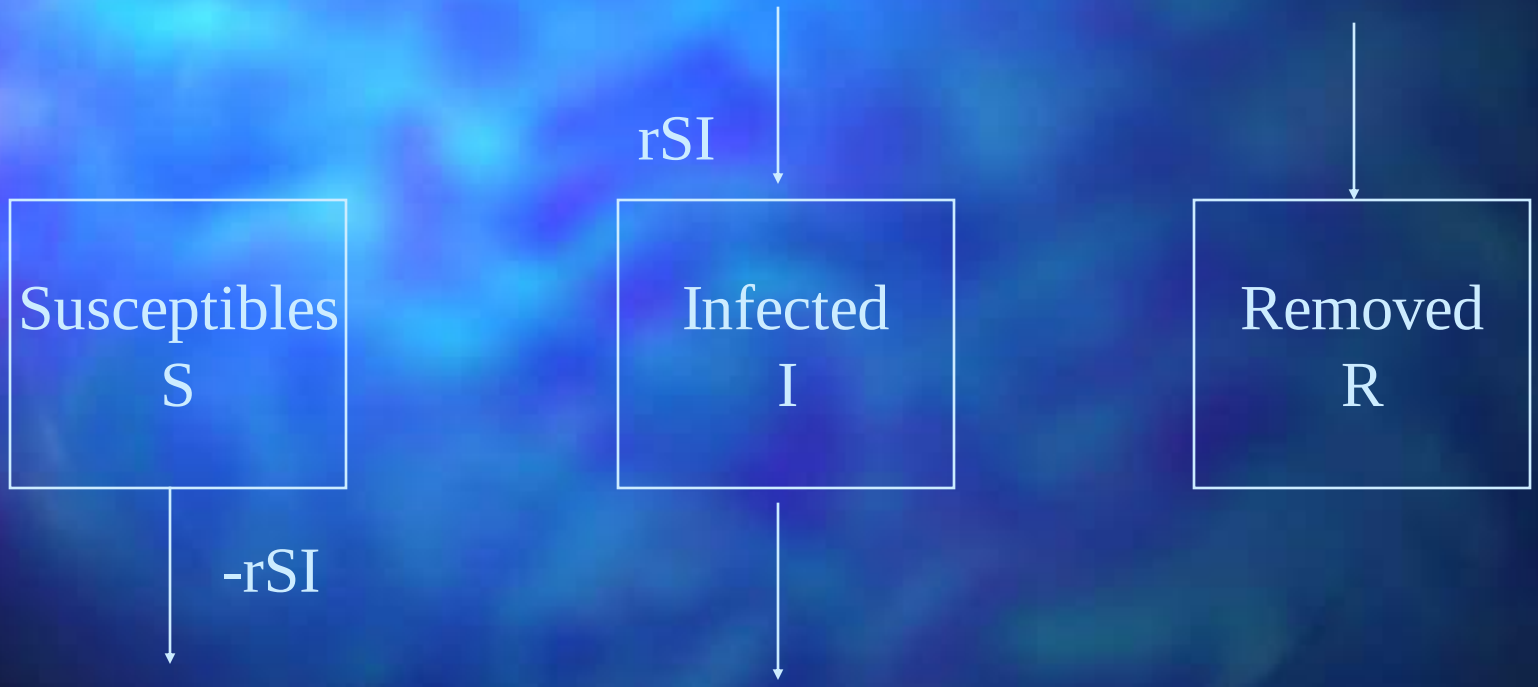

Where r is a constant and greater than 0.

This model also assumes that the incubation period of the disease “is short enough to be negligible.”

The rate of removal from the infectives into the removed class “is proportional to the number of infectives.”

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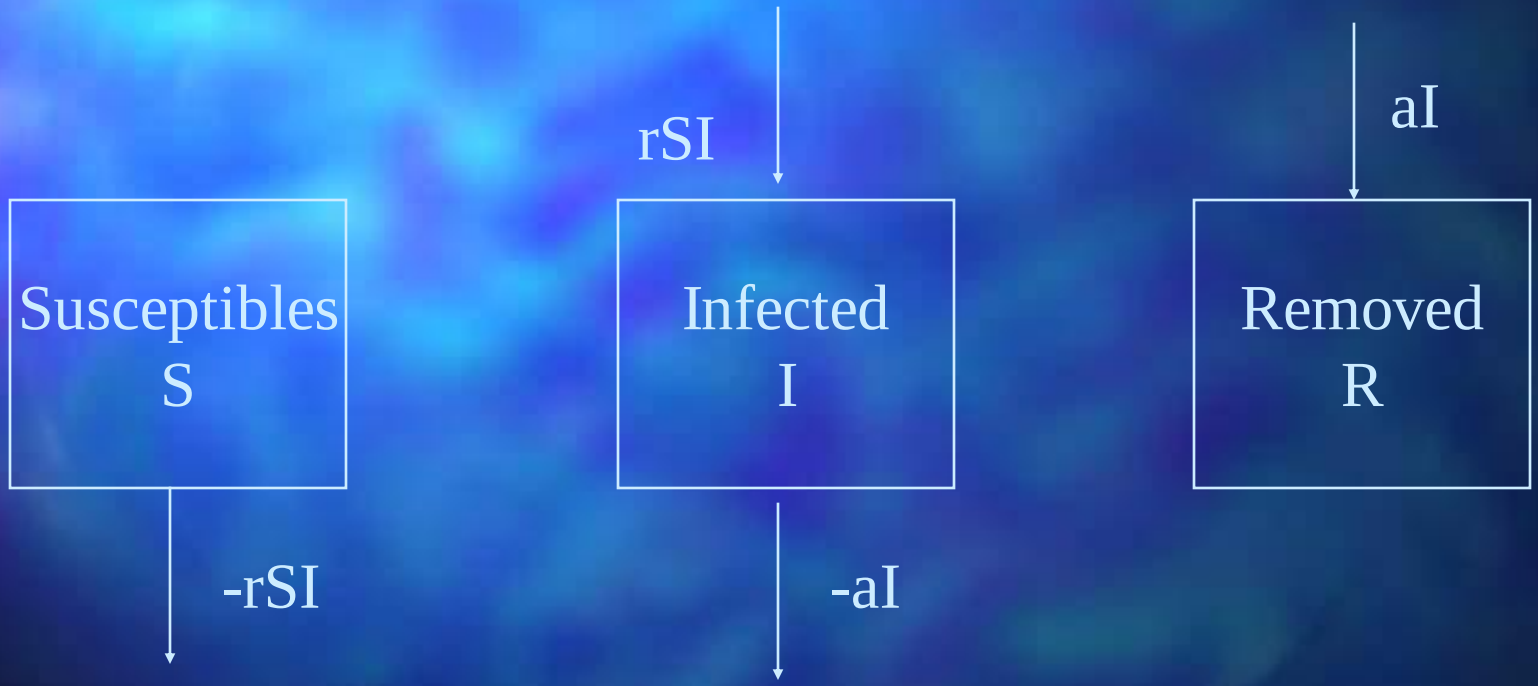

Where a is a constant and greater than 0.

“Assuming that every individual has equal probability of coming into contact with one another, the model becomes

$$\frac{dS}{dt} = -rSI$$

$$\frac{dI}{dt} = rSI - aI$$

$$\frac{dR}{dt} = aI$$

Samuel Harvey Fryer's Typhoid Fever Models

Causes of Typhoid Fever

A “bacterial infection of the intestinal tract and occasionally the bloodstream”

Caused by Salmonella Typhi

Spread by ingesting food or water that is contaminated with the human waste of an infected individual

Symptoms of Typhoid Fever

Fever

Headache

Constipation or Diarrhea

Enlarged Spleen and Liver

Assumptions Made by Fryer

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"The population is assumed to be uniform and homogeneously mixing in such a way that exposure to typhoid occurs equally and uniformly throughout the population."

Model 1

$$dS/dt = -aIS + bS$$

$$dI/dt = aIS + bS - fI$$

$$dR/dt = fI$$

where a is “an infectious contact rate”

b is rate of infective contact with the source

f is daily removal rate

Model 3

Susceptible

Exposed

Carrier

Infective

Removed

Model 3

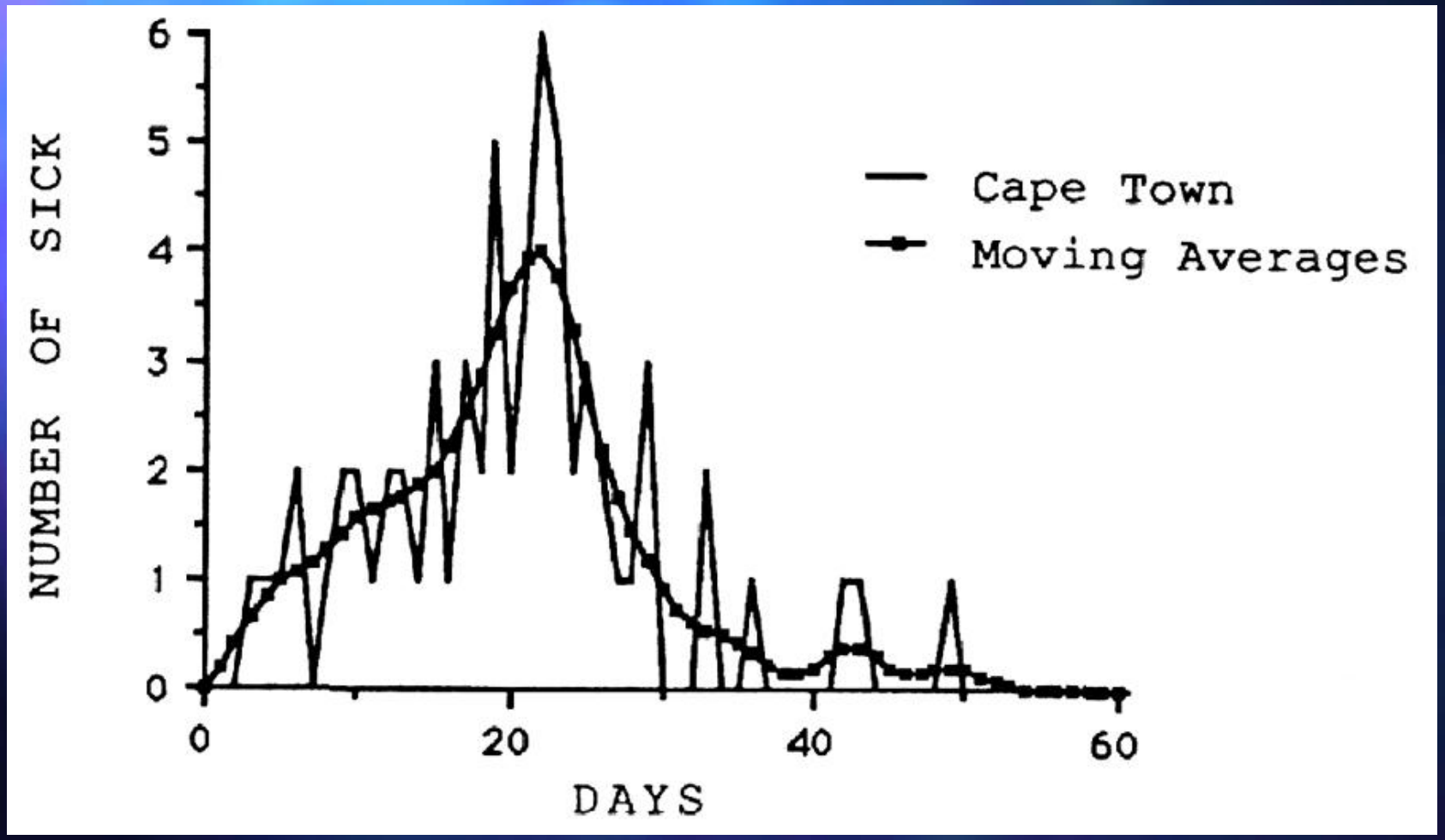
Movement Through Classes

S - E - C - R

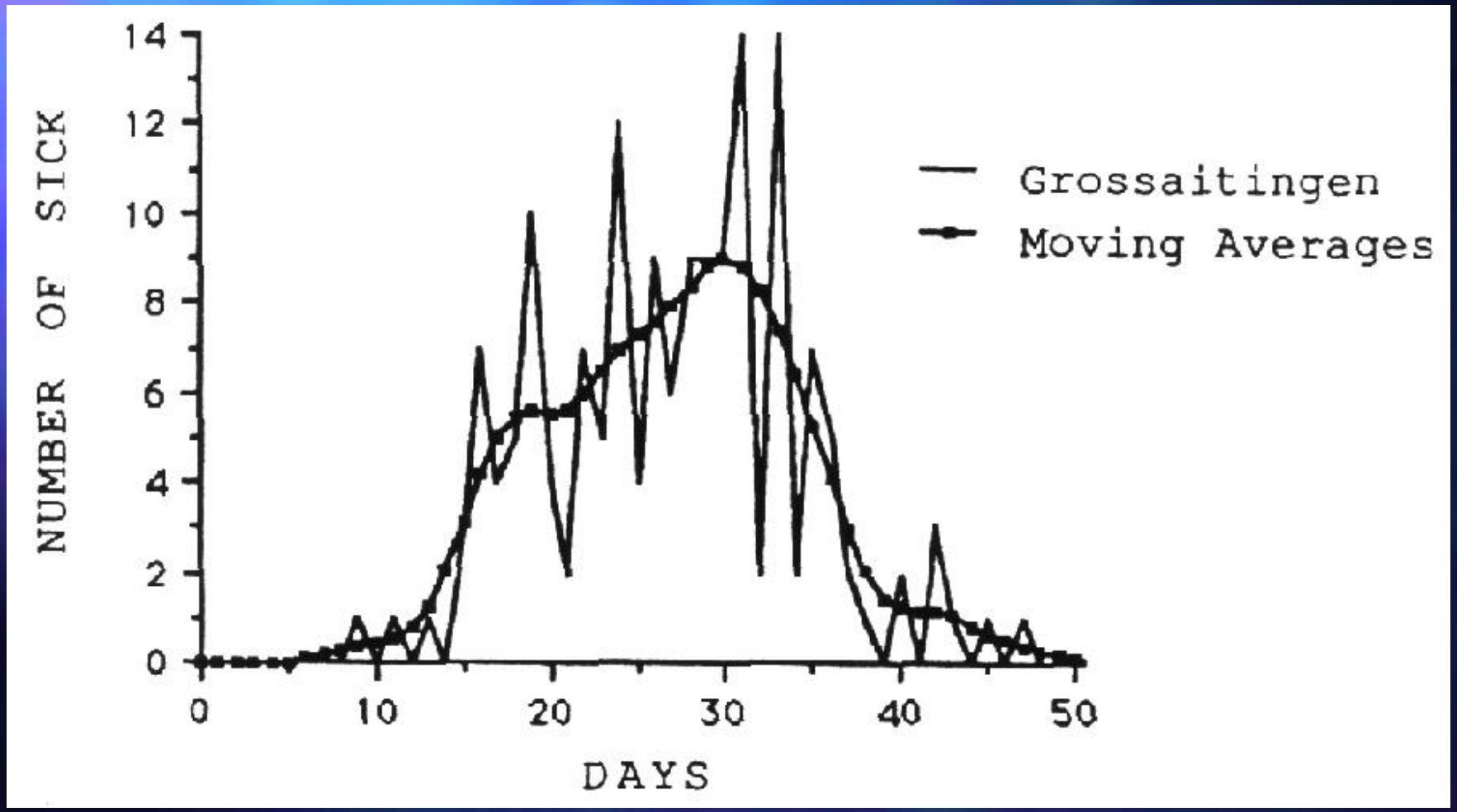
or

S - E - I - R

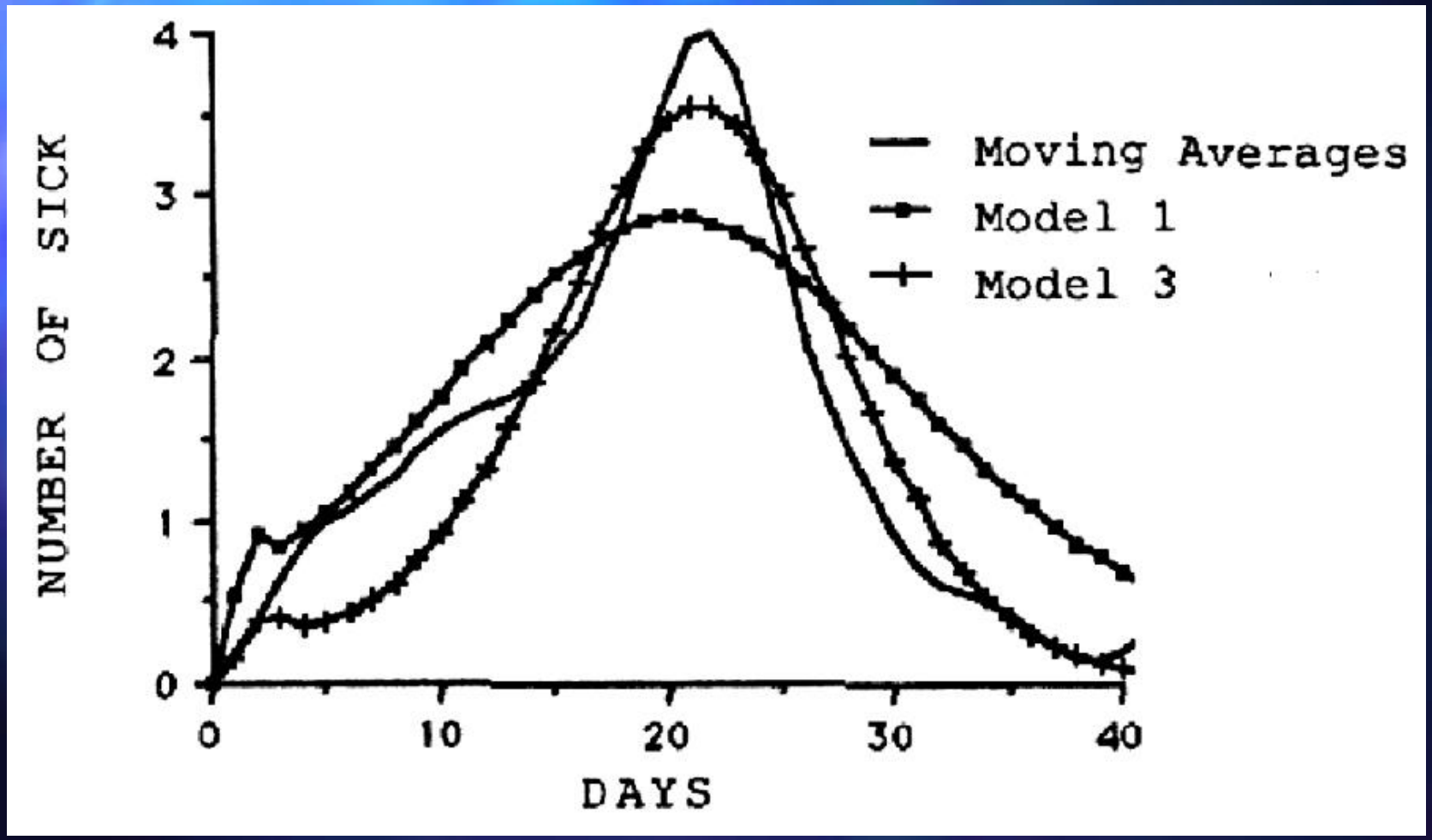
Actual Data of Cape Town Epidemic



Actual Data of Grossaitingen Epidemic



Model Comparison for Cape Town



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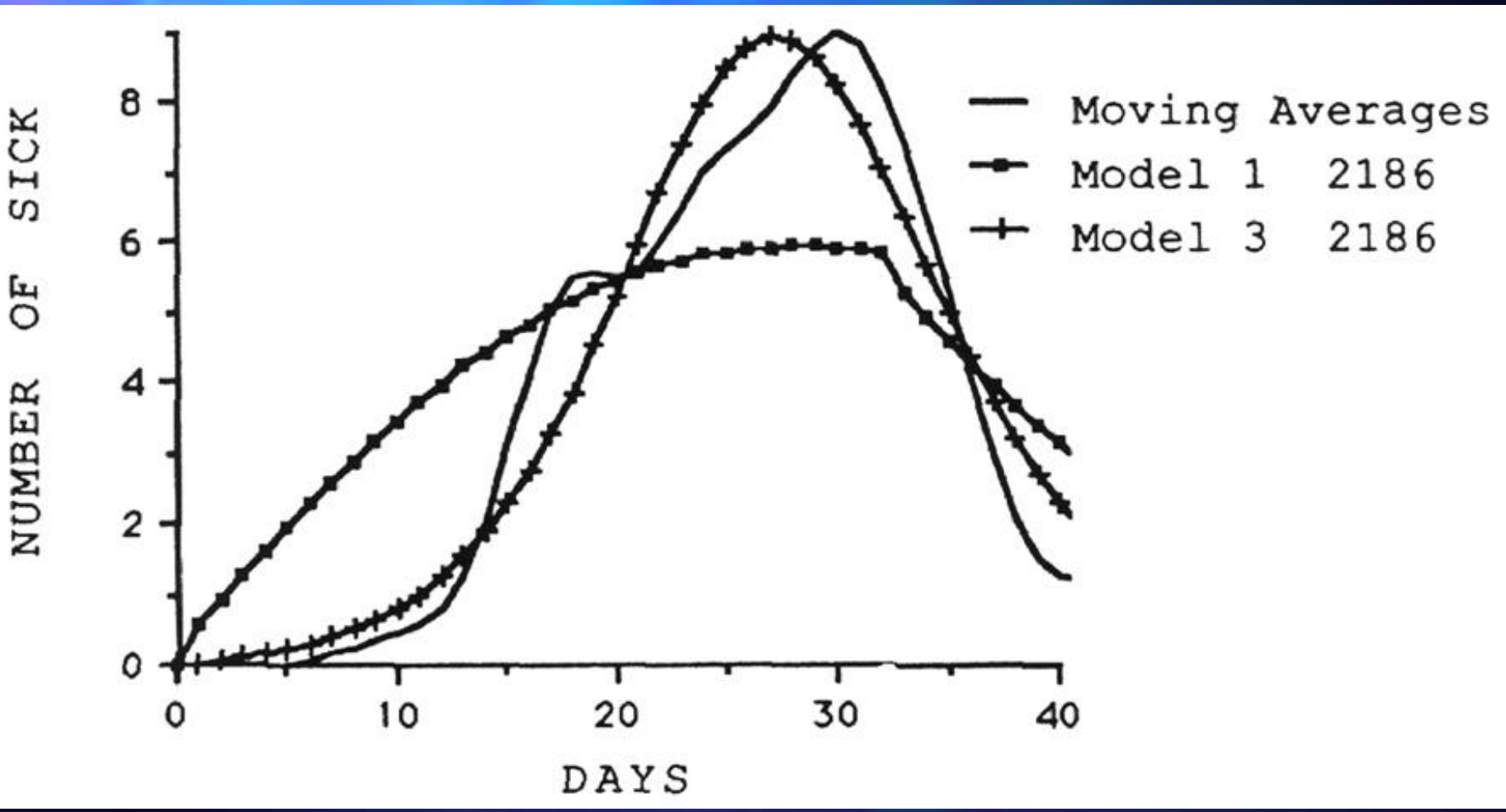

William H. Hamer's London Measles Epidemic Model

Causes of the Measles

Respiratory infection caused by a virus

Spread in fluid from the mouth and nose or airborne droplets

Symptoms of the Measles

Runny Nose

High Fever

Hacking Cough

Skin Rash

Two Classes for Hamer's Model

Susceptible

Infected

The following are assumptions made by Hamer

“There is a constant replenishment of susceptibles via newborns.” This “constant replenishment” is represented by a.

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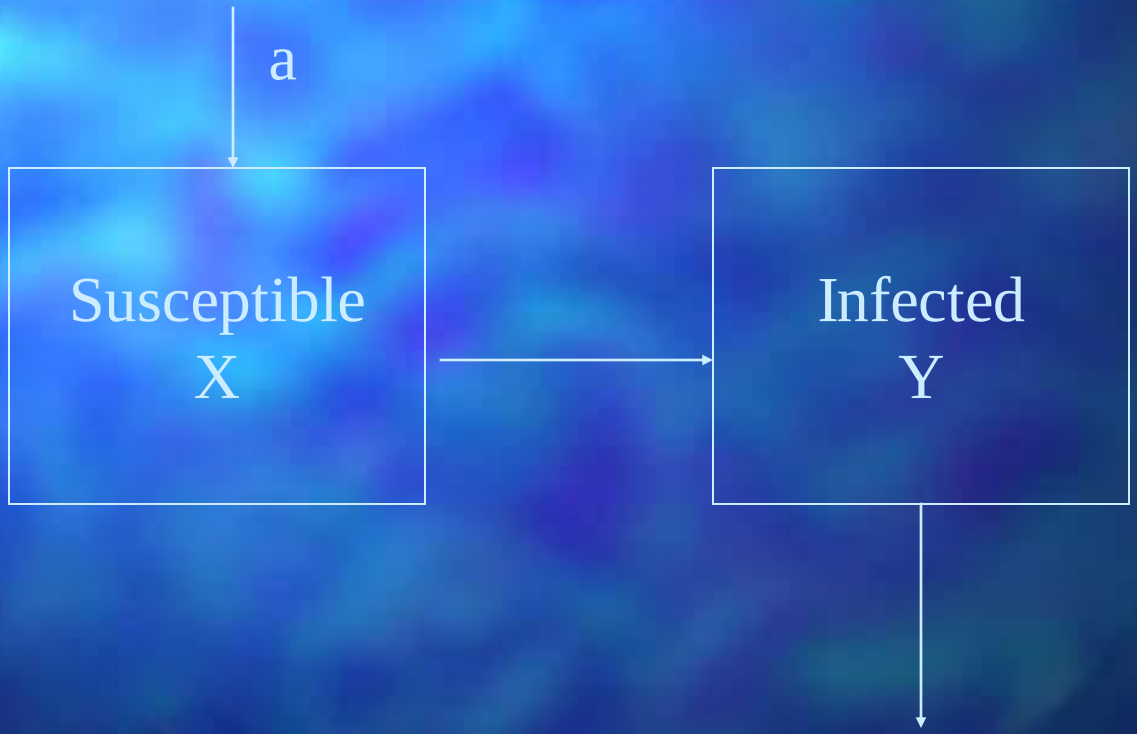

“The rate of new cases is jointly proportional to the number of susceptibles and the number ill.”

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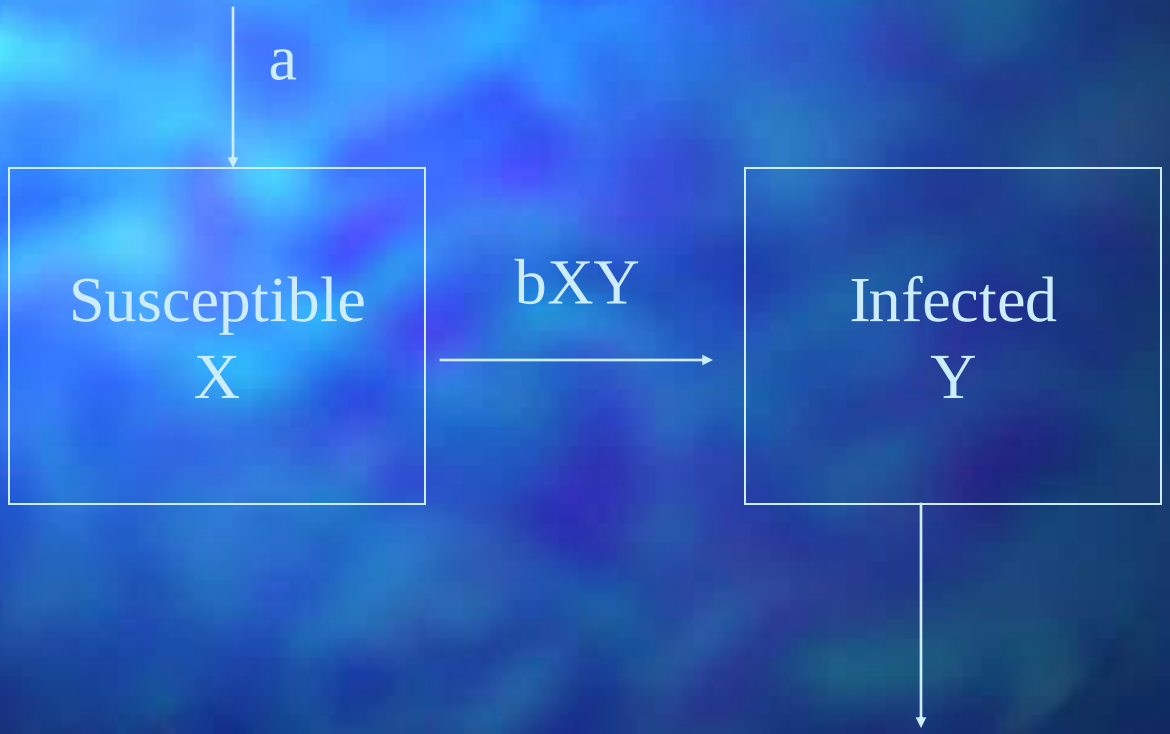

“The number of persons recovering was proportional to the number ill.”

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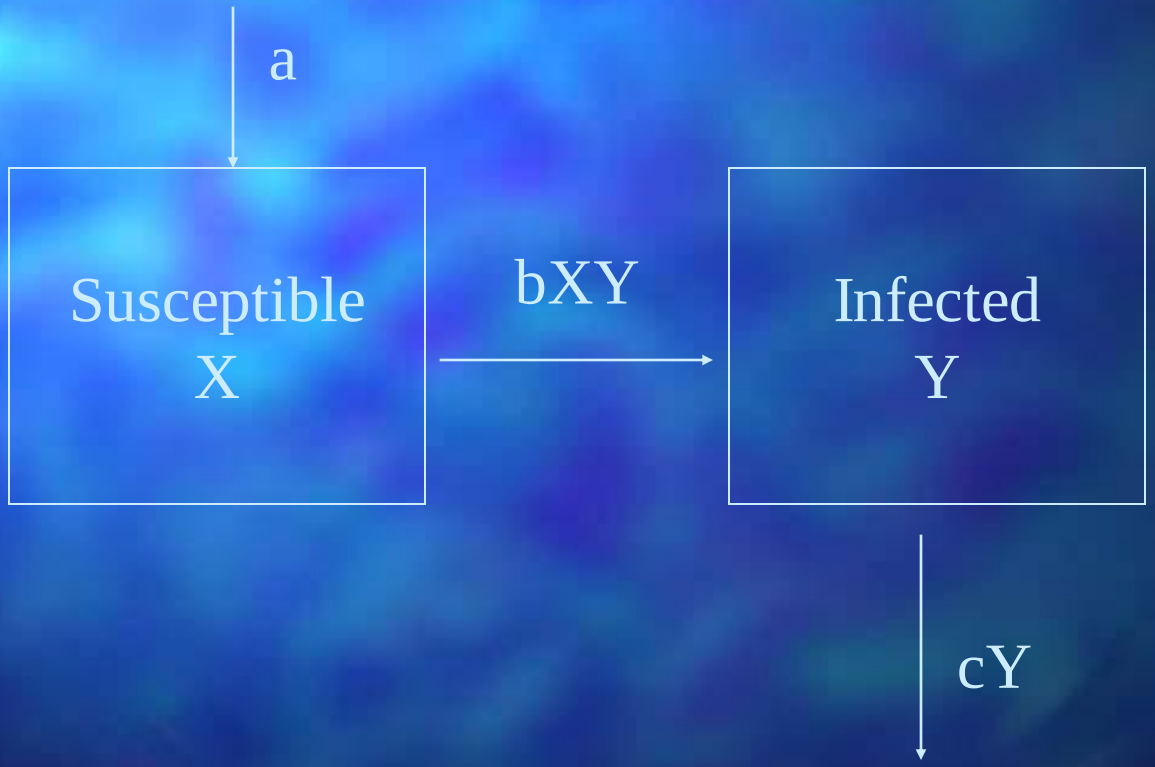


“Those not yet recovered were assumed equally contagious over the duration of the illness.”

“People who have recovered from a bout with measles are assumed to be immune and are no longer contributing to the spread of disease.”

Constants used in Hamer's Model

a is the rate of increase of the susceptibles

b is the rate of transmission

c is the removal rate

a, b, and c are all greater than 0.

Hamer's Measles Model

$$dX/dt = a - bXY$$

$$dY/dt = bXY - cY$$

Values for Constants

a = 2200

b = 1 / 300000

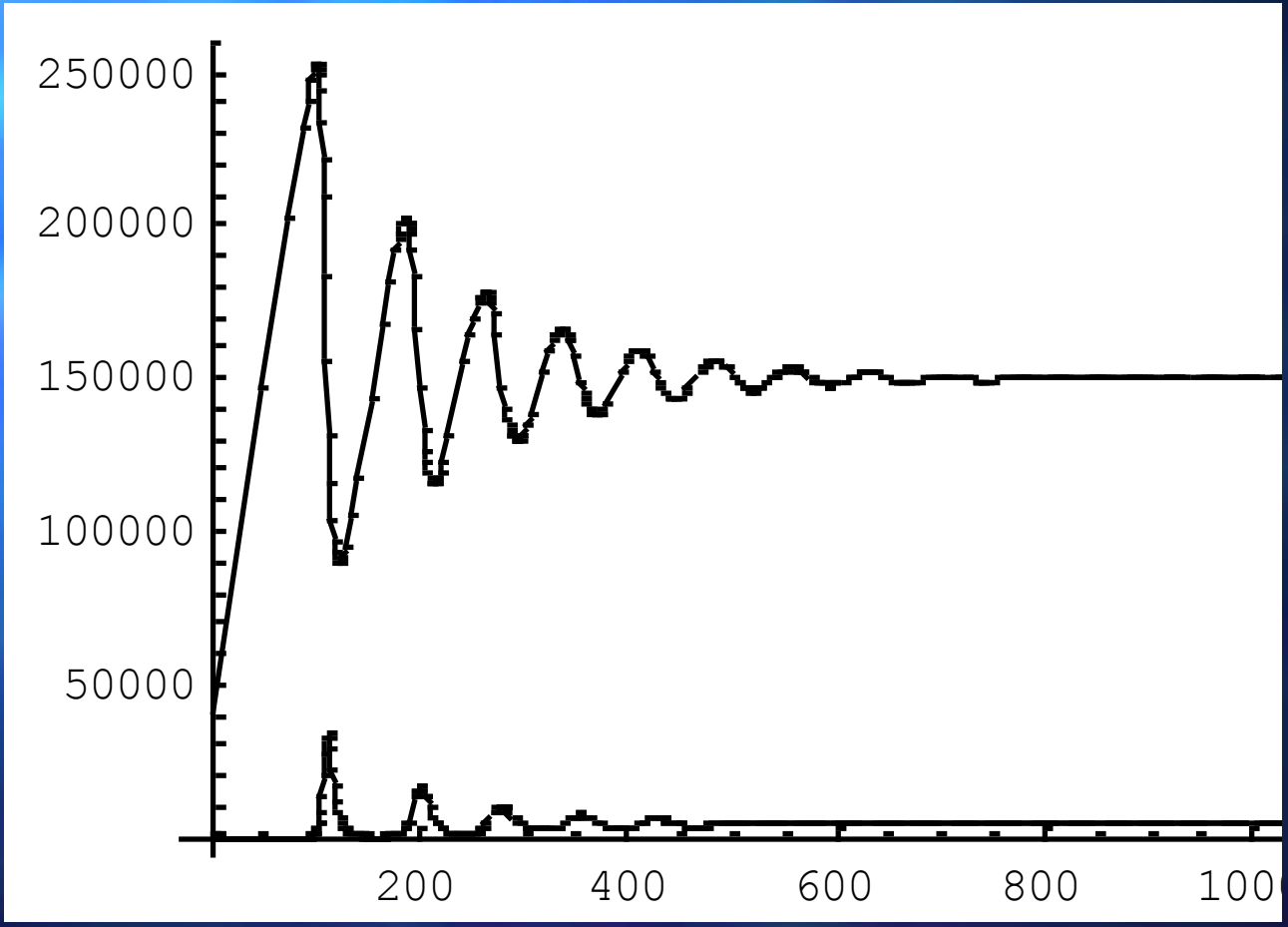
c = 1 / 2

Hamer's Measles Model

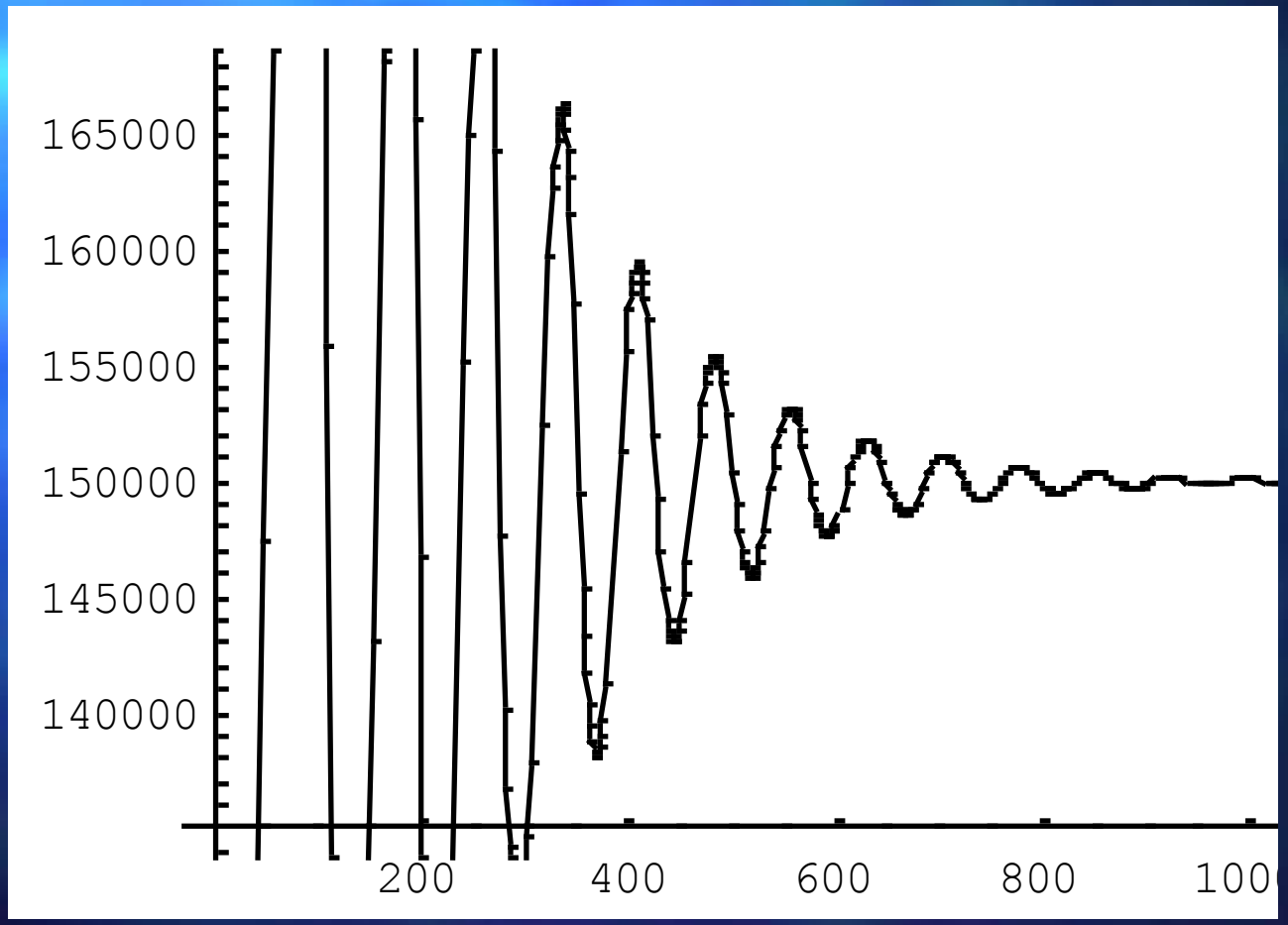
$$\frac{dX}{dt} = 2200 - \left(\frac{1}{300000}\right)XY$$

$$\frac{dY}{dt} = \left(\frac{1}{300000}\right)XY - \left(\frac{1}{2}\right)Y$$

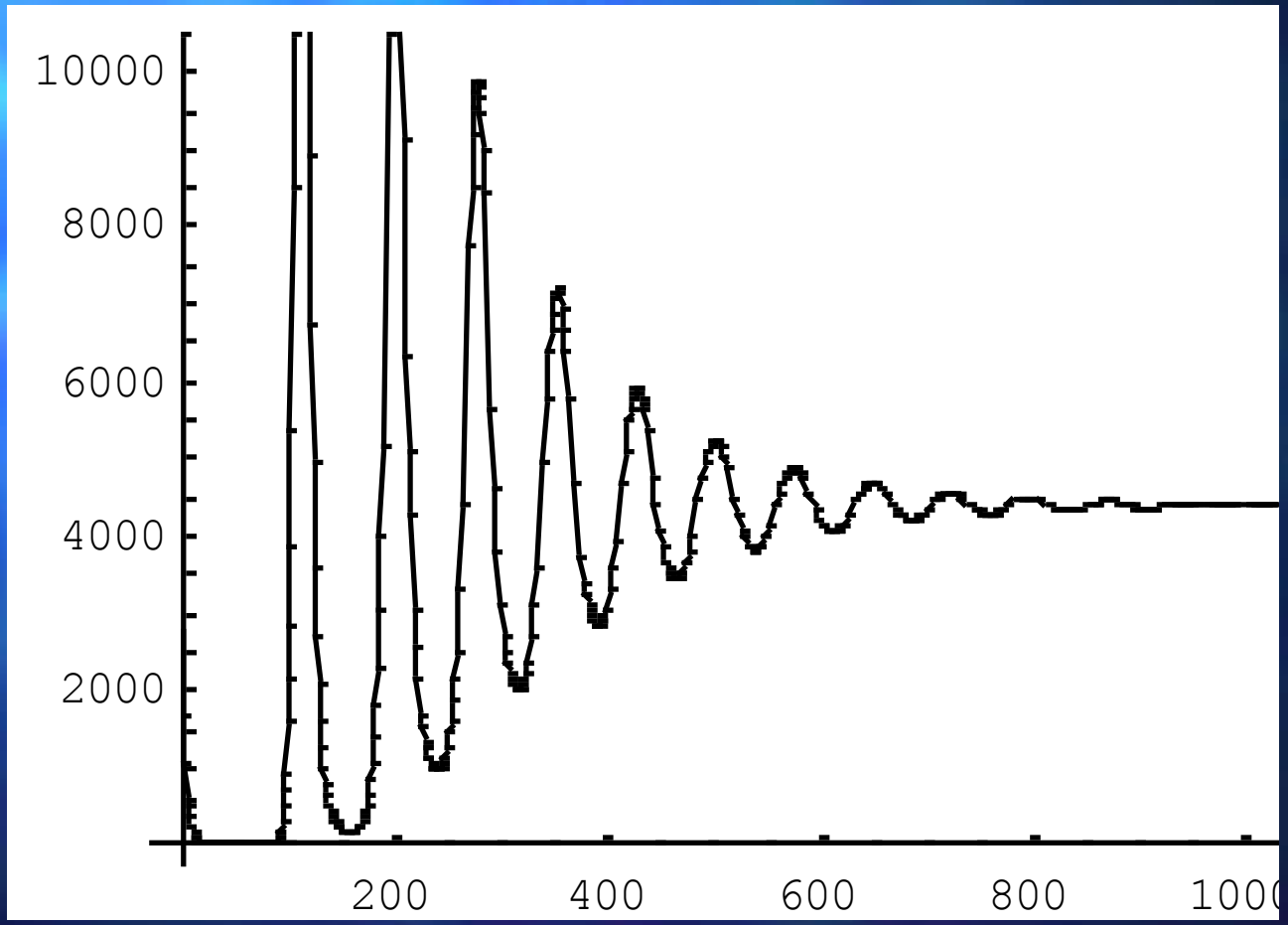
Solutions to Hamer's Model



Number of Suceptibles



Number of Infected



H. E. Sopher

H. E. Sopher

"…In this research I was merely following up the trail blazed by Sir William Hamer more than twenty years ago only in detail departing from his methods…"

Added Assumptions made by Sopher

Point infection law states “that newly infecteds must pass through an incubation period before they can transmit the disease and once this incubation period has passed, for all practical purposes they can transmit the disease only for an instant.”

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“All ill persons have same length incubation period” or an average incubation period.

Sopher's Model

$$y_{k+1} = b x_k y_k$$

$$x_{k+1} = x_k + a - y_{k+1}$$

Constants and Initial Conditions used by Sopher

$a = 1{,}000$

$b = 1 / 40{,}000$

$x(0) = 40{,}000$

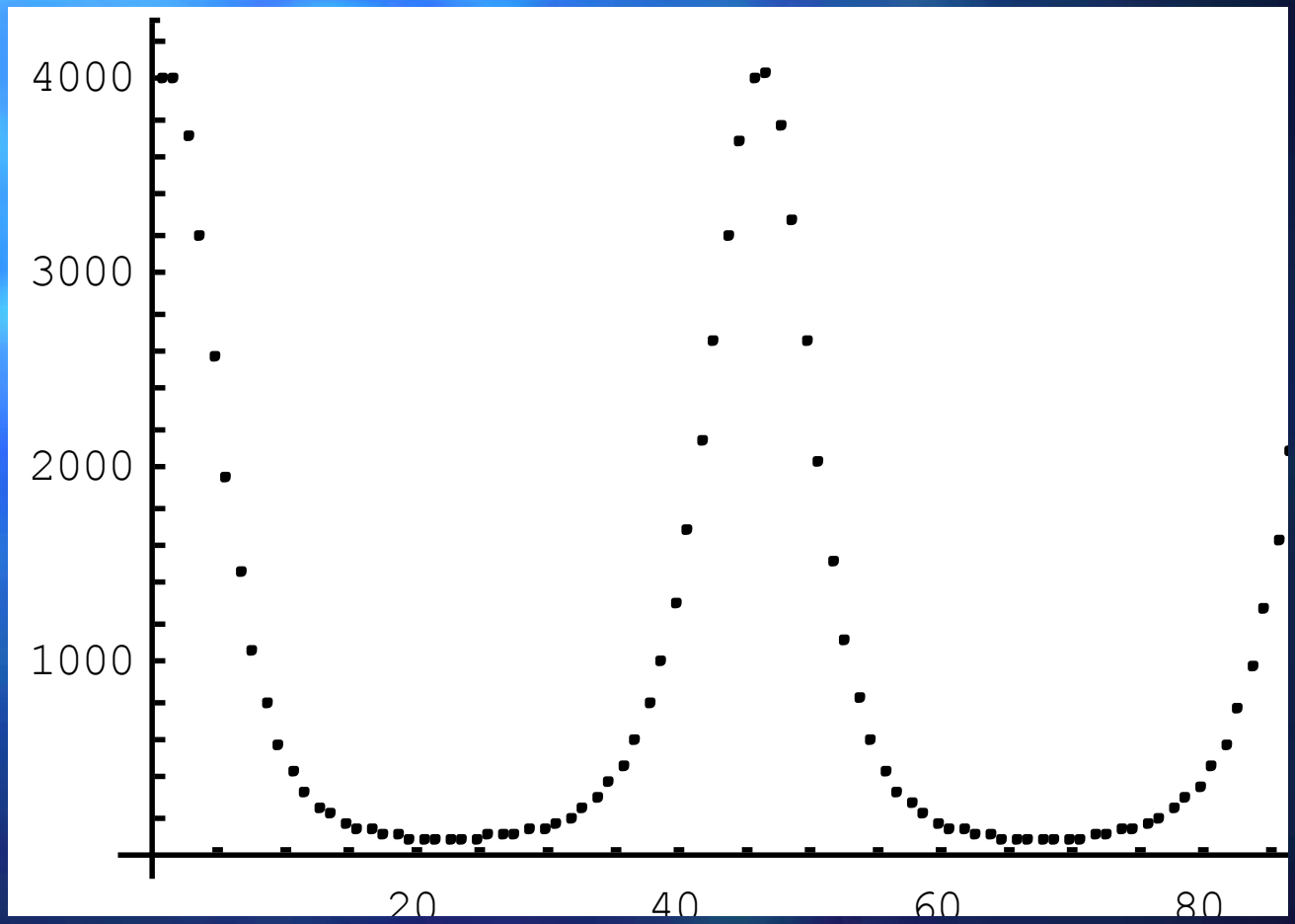
$y(0) = 4{,}000$

Sopher's Model

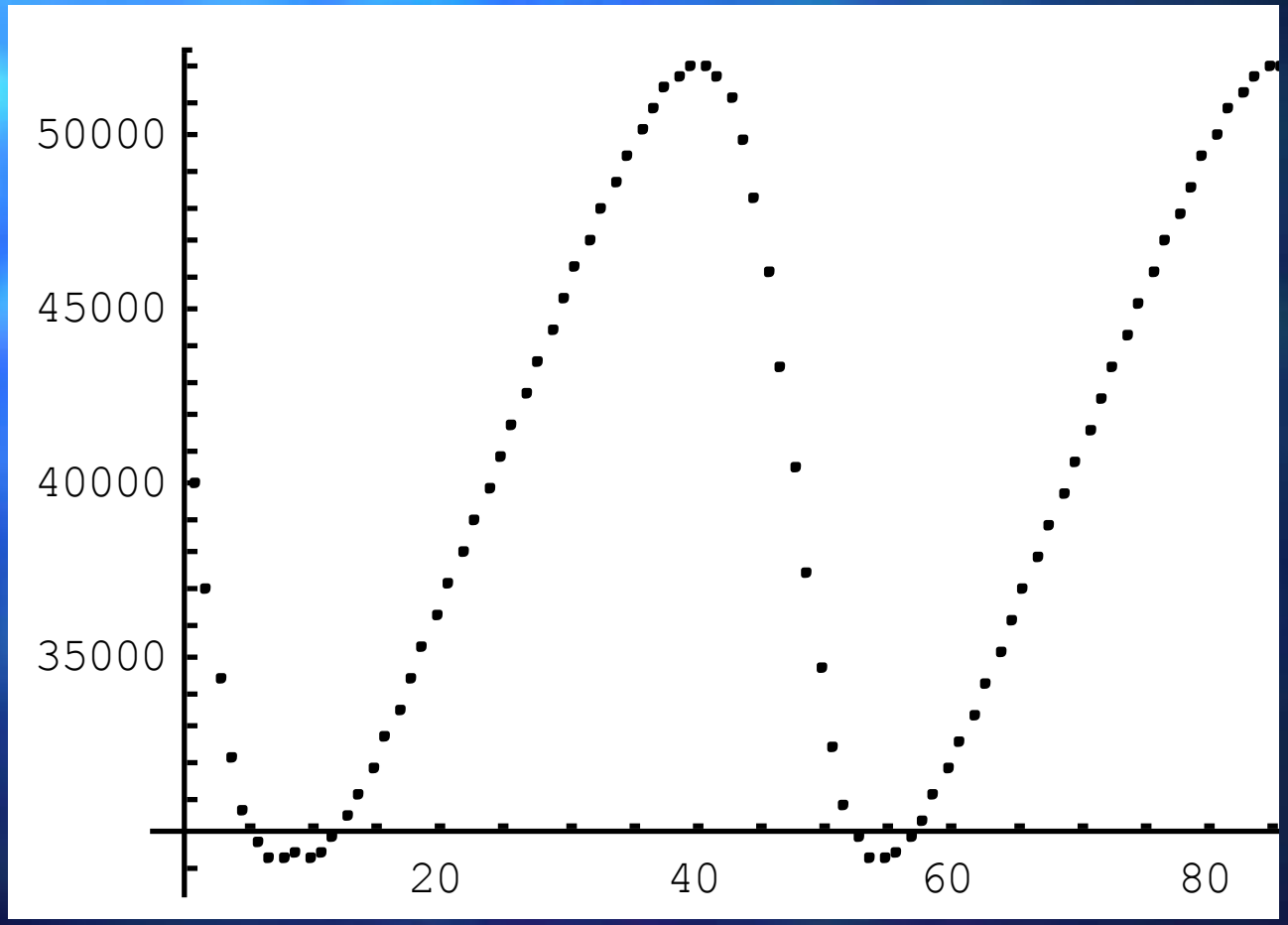
$$y_{k+1} = \frac{1}{40000} x_k y_k$$

$$x_{k+1} = x_k + 1000 - y_{k+1}$$

Number of Infected



Number of Susceptibles



These are just a few examples of epidemic models. There have been many done in the past and will be many done in the future to learn more about epidemics.