Determination of Liquid Viscosity Using a Bluetooth PASCO Smart Cart

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Introduction

A viscometer, or viscosimeter, is an instrument used to measure the viscosity of a fluid. The goal of this research project is to experimentally determine the resistance coefficients of viscous fluids and compare to accepted values measured by a viscometer.

A sphere of mass m is attached to a PASCO Bluetooth Smart Cart and submerged in a viscous fluid. The smart cart will act as our viscometer. The mass falls through the liquid when the cart is released from rest, and the velocity of the cart is applied with Stokes' Law to determine a value for the viscosity of the fluid.

Materials and Methods

This project is done by tying a string to a Bluetooth PASCO Smart Cart, running the string over a pulley, and attaching a metal sphere to the string. The sphere is submerged in a viscous fluid, such as pancake syrup or aloe. When the system is released from rest, it experiences linear drag force that acts on the falling mass.

The Bluetooth PASCO Smart Cart is a low-friction dynamics cart that can record its velocity over time and transmit this data to a computer via Bluetooth. It was connected to a sphere falling through viscous fluids to plot velocity versus time. The PASCO Capstone data-collection software fit this data with a theoretical function. This fit gives the viscosity. The average experimental viscosity is the average value of three trials for each fluid, and the standard deviation is based off the values for these trials (Ref. Data Table 1).

The fluids used were vegetable oil, pancake syrup, karo syrup, and aloe. Pancake syrup, karo syrup, and aloe have a maximum Reynolds number less than 10 in the experiments, thus implying laminar flow and large viscous forces. A maximum Reynolds number of less than 10 is required for Stokes' equation to be valid.

The metal sphere had a radius of R = 0.0122 m and a mass m = 0.0929 kg. A graduated cylinder was used to hold the fluid (1000 mL for aloe, 500 mL for all other fluids).

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Theory

The motion of a sphere in a fluid can be modeled with Stokes' Law. Stokes' Law assumes that flow is laminar or smooth. The determination of the Reynolds number is important because Stokes' Law implies a Reynolds number less than than 10. The Reynolds number is defined as:

$$Re = \frac{\rho R v}{\mu}$$

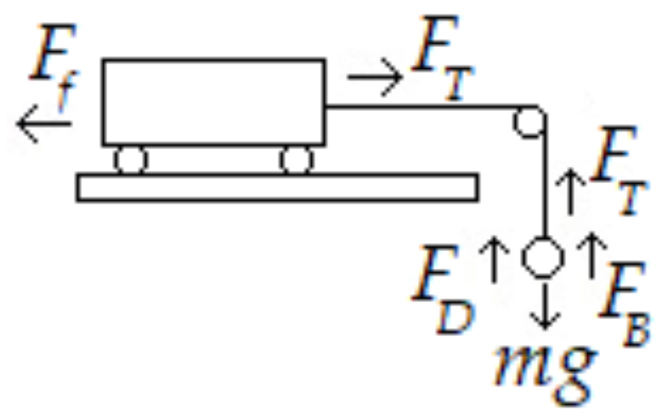
where ρ is the density of the fluid, R is the radius of the sphere, v is the velocity of the mass falling through the fluid, and μ is the viscosity of the fluid.

At low Reynolds numbers, flows tend to be dominated by laminar (sheet-like) flow, while at high Reynolds numbers flows tend to be turbulent. Laminar flow occurs at low Reynolds numbers, where viscous forces are dominant. Turbulent flow occurs at high Reynolds numbers and is dominated by inertial forces.

The application of Stokes' Law (1) for the drag force and the data collected from the smart cart allows for a determination of the viscosity of the chosen fluid.

$$F = 6\pi R\mu v \qquad (1)$$

The force diagram below of the cart plus sphere is used to find the sum of the forces acting on both the submerged mass (2) and the smart cart (3).



$$mg - F_T - F_B - 6\pi R\mu v = ma \qquad (2)$$

$$F_T - F_f = m_c a \qquad (3)$$

The sum of these forces reduces to one equation (4).

$$mg - F_B - F_f - 6\pi R\mu v = (m + m_c)a \qquad (4)$$

A separation of variables is required to solve for the velocity, v. The following steps are a derivation of the velocity.

$$\frac{dv}{dt} = \left(\frac{mg - F_B - F_f}{m + m_c}\right) - \left(\frac{6\pi\mu R}{m + m_c}\right)v$$

$$\frac{dv}{dt} = \mathrm{A} - \mathrm{B}v$$

$$A = \left(\frac{mg - F_B - F_f}{m + m_c}\right) \qquad B = \left(\frac{6\pi\mu R}{m + m_c}\right)$$

$$\frac{dv}{A - Bv} = dt$$

$$\int_{t_0}^{t} -dt = \int_{v_0}^{v} \frac{dv}{Bv - A}$$

$$v = \frac{mg - F_f - F_B}{6\pi\mu R} + \left(v_0 - \left(\frac{mg - F_f - F_B}{6\pi\mu R}\right)\right)e^{-B(t-t_0)}$$

Using the PASCO Capstone velocity versus time graph, each run was fit as:

$$v = C\left(\frac{1}{A}\right) + \left(v_0 - C\left(\frac{1}{A}\right)\right)\left(e^{-D\,(t-t_0)}\right)$$

$$C = \frac{mg - F_f - F_B}{6\pi R} \qquad D = \frac{6\pi R}{m + m_c}$$

$$F_B = \rho_f\left(\frac{4}{3}\pi R^3\right)g$$

where C and D are constants. C is dependent upon the buoyant force F_B for a particular fluid. The buoyant force was found for each liquid (ref. Table 1). The friction force F_f is assumed to negligible due to the smart cart being low-friction. The variable A is the viscosity for the chosen fluid, which PASCO Capstone software fits a value for.

Data (Table 1)

Fluid	Maximum Reynolds Number	Buoyant Force (N)	Temperature (°C)	Accepted Viscosity (Pa·s)	Average Experimental Viscosity (Pa·s)	Standard Deviation (Pa·s)
Vegetable Oil	12.57	0.0685	24	0.125	0.2022	±0.046
Pancake Syrup	7.07	0.0981	22	2.5	4.45	±0.054
Karo Syrup	0.573	0.1020	24	5.0	20.5	±1.2
Aloe	0.3253	> 1	21	9.073	3.018	±8.94

Discussion and Results

The experimental viscosities for vegetable oil, pancake syrup, and karo syrup are higher than the compared accepted values. Vegetable oil and pancake syrup performed better than karo syrup and aloe. Their experimental viscosities and the standard deviation between them was closer to the expected value than the fluids with Reynolds numbers less than 1.

Aloe is the outlier in this experiment, as it possesses the lowest Reynolds number, but the experimental viscosity was smaller than the expected value. Aloe also had the largest standard deviation.

The maximum Reynolds number of both pancake syrup and karo syrup is less than 10, implying a laminar flow in this experiment. Vegetable oil has a maximum Reynolds number larger than 10, but the standard deviation showed that it performed as well as the other fluids experimentally.

The large standard deviation between the experimental viscosities of aloe show that the viscosity of aloe varies experimentally, which could be due to the large viscosity and the bubbles present in the fluid, creating additional conditions to consider.

Conclusion

Vegetable oil had a maximum Reynolds' number of 12.57, and its viscosity was within a factor of 2 of the accepted value. The maximum Reynolds number must be less than 10 for laminar flow and the validation of Stokes' Law. For the case of vegetable oil, Stokes' Law still provides a good approximation for the viscosity. For this experiment, a maximum Reynolds number not much larger than 10 could be used if most data points included are for an Re less than 10.

There is uncertainty in the maximum Reynolds number. The environment plays a large part in the determination of the experimental viscosity because the viscosity of the chosen fluids is affected by temperature. Therefore, varying temperature experimentally could impact vegetable oil's maximum velocity, thus changing the Reynolds number.

This experimental approach for finding the viscosity with a PASCO Bluetooth Smart Cart gives values within a range of 2-4 times the accepted values. The standard deviation between these values is small, except for aloe. Aloe possesses physical properties that could contribute to the error in measurement.

Sample Data with Curve Fit