MEASURING THE CHAOTIC BEHAVIOR OF DRIPPING WATER

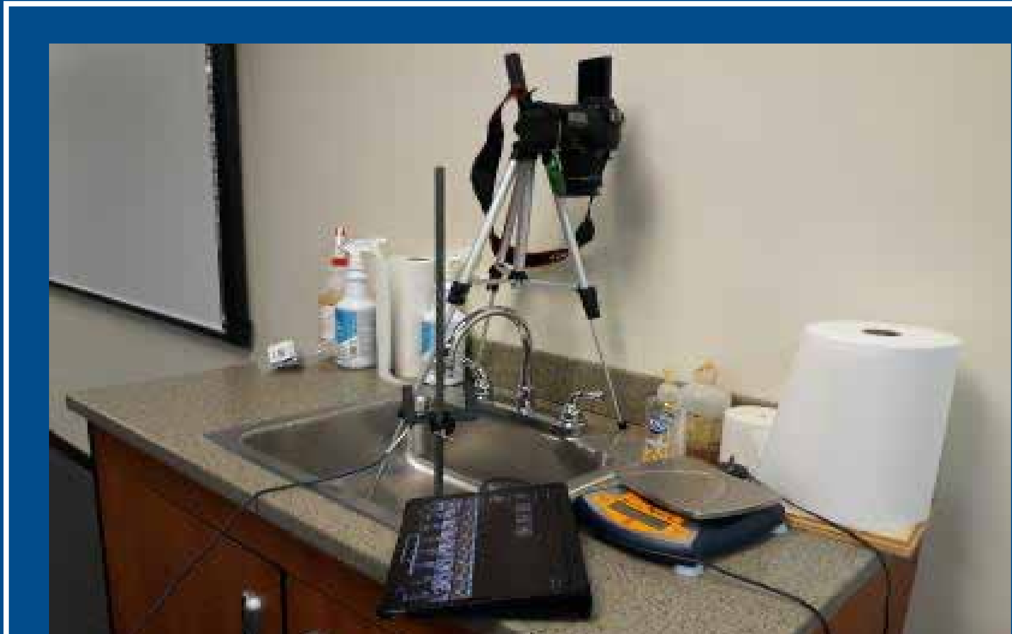
Presentation By: Scott Morris
Acknowledgments: Dr. G and Ashley Perkins

INTRODUCTION

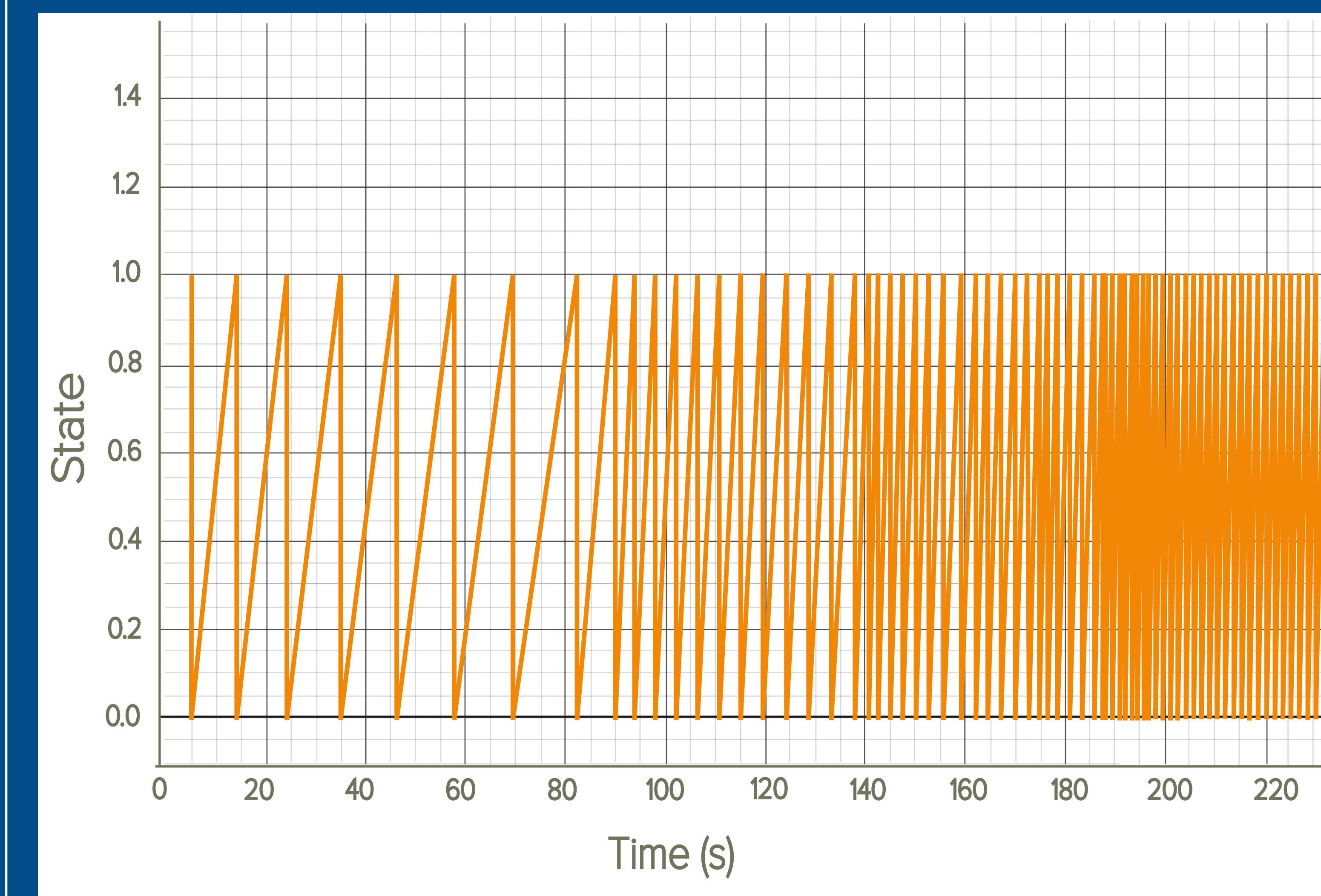
The focus of this project is to measure the chaotic behavior of water droplets under various conditions and, if possible, to use these measurements to calculate the Feigenbaum bifurcation constant, δ. According to several published works, the driprate of water does not smoothly and continuously increase over time in response to increasing water pressure. Instead, the falling droplet's frequency sharply doubles over distinct intervals until the system eventually becomes chaotic. By adopting and utilizing several different experimental techniques, this research chronicles the efforts in documenting this odd, counterintuitive behavior.

THEORY OR METHODS

Throughout the course of this experiment, several different experimental techniques were utilized to measure the Feigenbaum bifurcation constant. Initially, I attempted to carefully, manually adjust a faucet handle to produce frequency doubling in a typical faucet's driprate. A Photogate Head was mounted below the faucet head, allowing for the direct measuring of the driprate. A laser is fired from the Photogate Head, and whenever it becomes blocked (due to a falling droplet of water), a signal is sent to a computer. These signals are then graphed as ones and zeros in Capstone. Additionally, a Canon EOS 80D camera was mounted above the faucet handle, allowing for the measurement of the faucet handle's change in angle. The purpose of this was to determine if the formula $\lim_{n\to\infty}\left(\frac{a_{n-1}-a_{n-2}}{a_n-a_{n-1}}\right)=\delta$ (or, rather, a finite termination of the formula) could be applied to the arclengths characterized by the



EXAMPLE OF FREQUENCY DOUBLING



transitionary angles of the faucet handle to approximate the Feigenbaum bifurcation constant. Secondly, I tied one end of a string to a small pole and tied the other end to the faucet handle. I then slowly rotated the pole about its symmetrical axis of rotation, winding the string around the pole, causing the faucet handle to turn. Whenever the driprate would transition, I would mark the location of the string's current position relative to the pole with a pen. Like Procedure One, this was done to determine in the formula $\lim_{n\to\infty}\left(\frac{a_{n-1}-a_{n-2}}{a_n-a_{n-1}}\right)=\delta$ could be applied to the lengths of the sections of the string characterized by the transitionary angles of the faucet handle. A Photogate Head was also placed below the faucet head to directly monitor the driprate. Thirdly, I attempted to construct several different variations of a draining reservoir. A container filled with water was attached to a thin capillary tube of 10mm diameter. This was chosen in hope that the resulting capillary action would directly oppose the draining water, preventing a constant outpouring of water. Several different iterations of this procedure (about half a dozen) were conducted to ensure consistent results.

RESULTS

Using the first method, period doubling was observed, but the transitional points were extremely sensitive on the angle of the faucet handle. Whenever the driprate transitioned, snapshots were taken of the faucet handle's corresponding angle. The changes in the transitional angles ranged from 0.604° to 0.691°, an extremely small window for reliable data analysis. Using the second method, period doubling was observed. However, the demarcations proved to be far too narrow, sometimes creating overlap in the penstrokes. The third method proved to be unreliable, regardless of iteration. Under some iterations, the driprate would never decrease. Under other iterations, the driprate would decrease, but would do so smoothly and continuously. Sometimes, instead of the driprate suddenly halving (which would be expected from a draining reservoir), the frequency would suddenly double.

CONCLUSIONS

Out of every tested method, only two exhibited period doubling, and out of those two, only one method yielded quantifiable results for analysis. From the graph provided, the driprate transitioned from an average of 0.092Hz, 0.225Hz, 0.404Hz, and 0.594Hz, showing a rough yet noticiably distinct correlation with the predicted period doubling. Using the transitionary angles for the a_n, the Feigenbaum constant was measured to range between 0.437 and 0.874, a significant departure from its known value. It should be noted that the Feigenbaum constant seems to be notoriously difficult to measure. While I found other research which documents frequency doubling in dripping faucets, I could not find any which has been able to accurately measure the Feigenbaum constant.

FURTHER PLANS

Regrettably, due to health concerns, I was unable to reach this experiment's full potential. If time constraints were not an issue, then I would have liked to have discovered a consistent method for producing frequency doubling in a dripping faucet, as well as a method for easily extrapolating the results of the experiment. Various other methods of experimentation were considered (such as constructing my own sink with an extremely sensitive facuet), but these plans were unable to be realized due to various reasons.